A game between bounded rationals using Genetic Algorithm

Does it pay to be “more rational”?

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Bounded rationality

Why economists have long expressed dissatisfaction with the full rationality assumption that is so pervasive in economic theory?

- Most economic agents are not in fact maximizers
- Many maximization procedures are very difficult to carry out in practice
- Experiments show that people often fail to conform to basic conclusions of rational decision theory
- Results of rational analysis sometimes seem unreasonble even on the basis of simple introspection.

Does it pay to be “more rational”?

Not all the time!

Under full-rationality: There is no issue as to who is more rational

Under bounded rationality: It is an unavoidable question

Who is more rational?

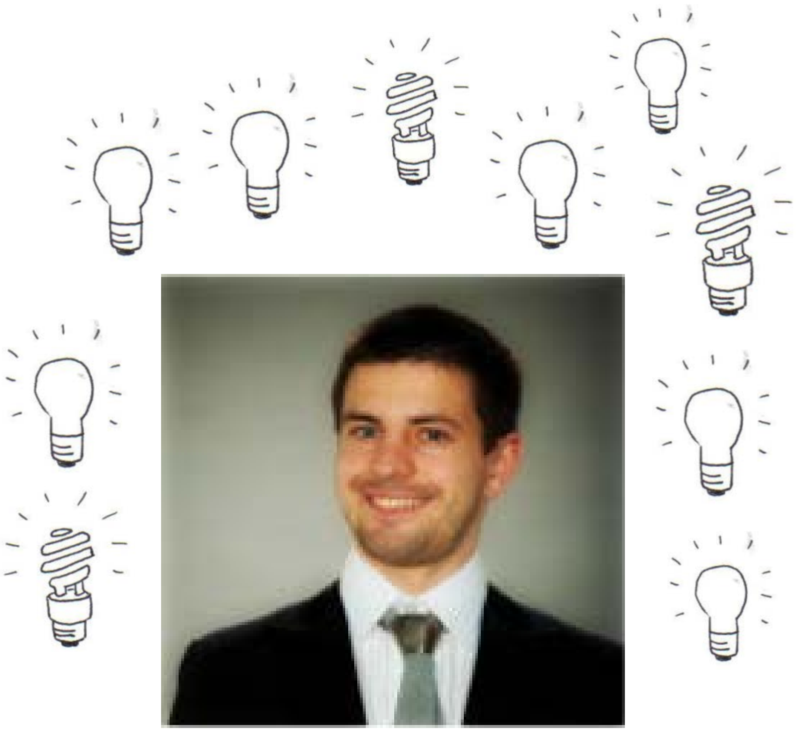
Abe

93% rational

Bob

65% rational

Who is more rational?



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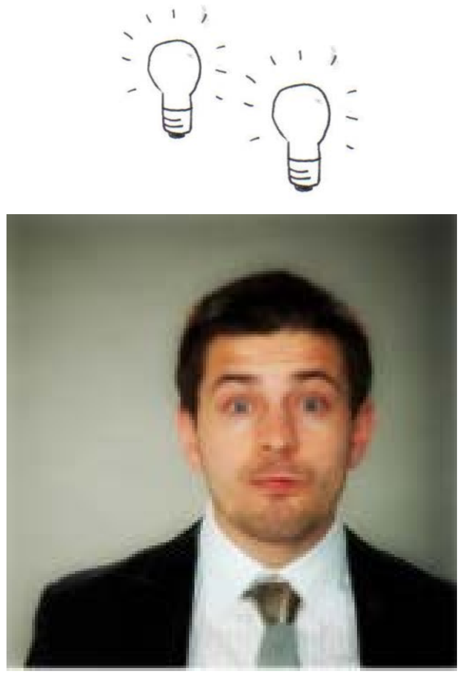


Bob

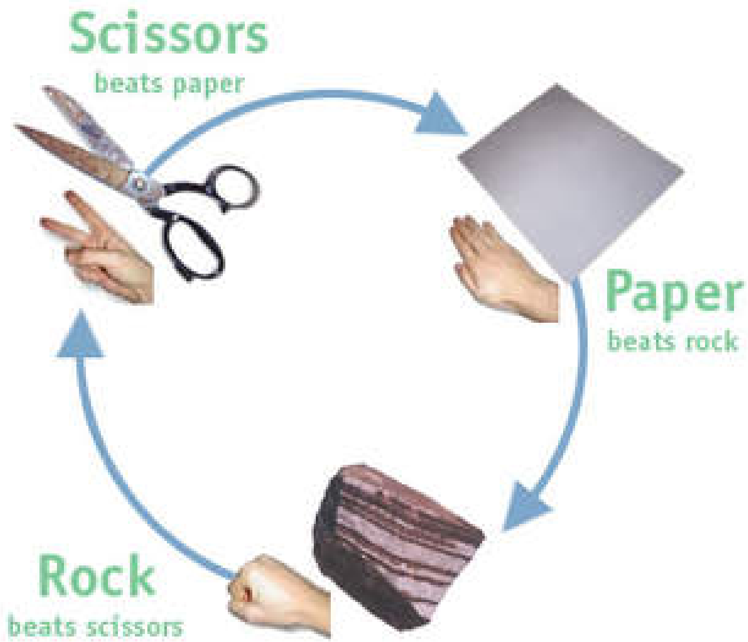
- Can more information really lead to higher payoff?

Assumptions:

- No constraint on computability capacity of individuals (i.e. no information overload or infoxication)
- Marginal utility from additional info is positive and constant
- Each information is distinct and has the same quality (i.e. no info is superior than the other)
- Information sets are tight (i.e. there is no superfluous info in the information set of agents)

All other things constant, we want to see only the effect of the size of the information endowment of an individual on her payoff.

Rock-Paper-Scissors Game



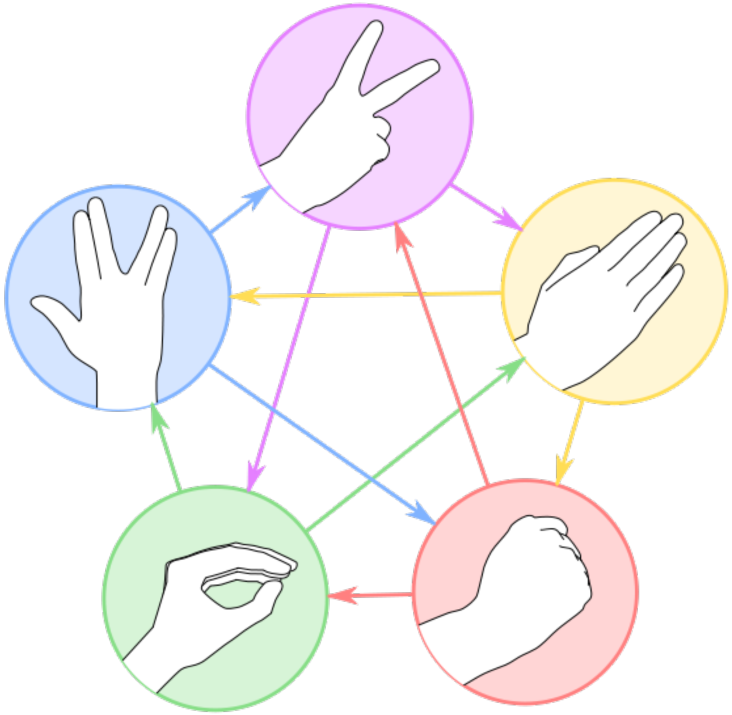
Player 2

Player 1

	R = 1/3	P = 1/3	S = 1/3
R = 1/3	0 , 0	-1 , 1	1 , -1
P = 1/3	1 , -1	0 , 0	-1 , 1
S = 1/3	-1 , 1	1 , -1	0 , 0

Nash Equilibrium: [(1/3, 1/3, 1/3) , (1/3, 1/3, 1/3)]

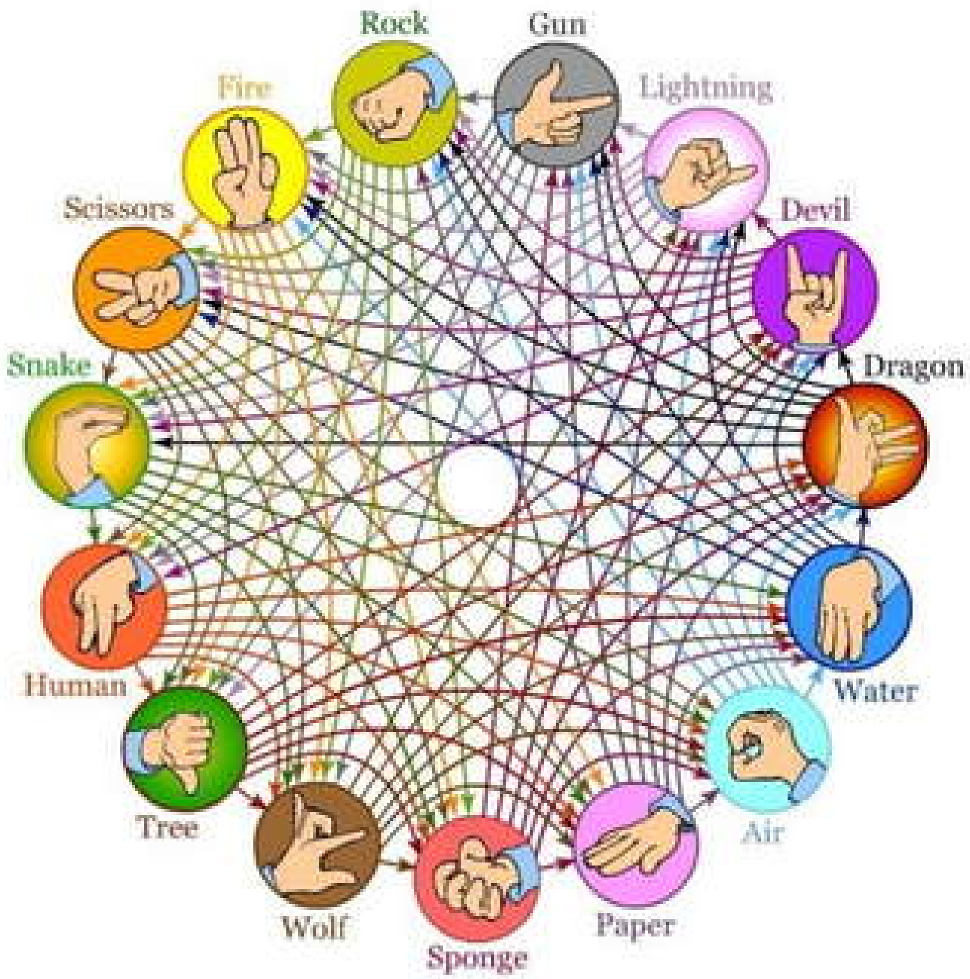
Rock-paper-scissors-lizard-spock



Each option:

(i) can defeat two other options and

(ii) can be defeated by another two



For n=15, each option can defeat 7 and can be defeated by 7 other

Literature:

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- Jiang , L et al. (2011), “Effects of competition on pattern formation in the rock-paper-scissors game”. *Pysical Review E*.
- Miller, John (1996), “The coevolution of automata in the repeated prisoner’s dilemma”, *Journal of Economic Behavior & Organization*.
- Ali, F and Nakao Z. (2000), “Playing the Rock-Paper-Scissors game using Genetic Algorithm”, *IEEE.*
- Gilboa, I and D. Samet (1989), “Bounded Rationality vs. Unbounded Rationality: The Tyranny of the Weak” , *Games and Economic Behavior*.

A. Both are unbounded:

Five-information universe

Player 1		Player 2
Rock, Lizard, Paper, Spock, Scissors	vs	Rock, Lizard, Paper, Spock, Scissors

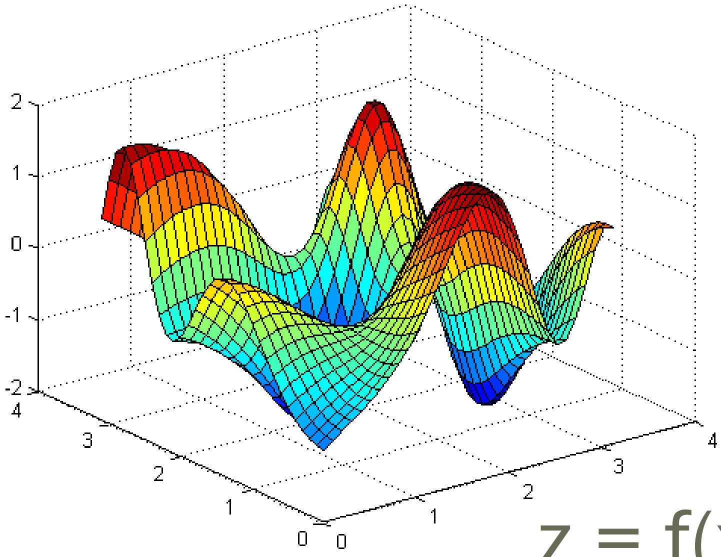
B. One is unbounded, the other bounded:

Player 1		Player 2
Rock, Lizard, Paper, Spock, Scissors	vs	Rock, Lizard, Paper

C. Both are bounded:

Player 1		Player 2
Rock, Spock	vs	Rock, Lizard, Paper

Optimization:



$z = f(x, y)$

$\pi_x = f(x_1, x_2, x_3; y_1, y_2, y_3)$

$\pi_x = f(R, P, R; S, S, S)$

$\pi_x = 2 \quad \pi_y = 1$

f is qualitative

Genetic Algorithm (GA)

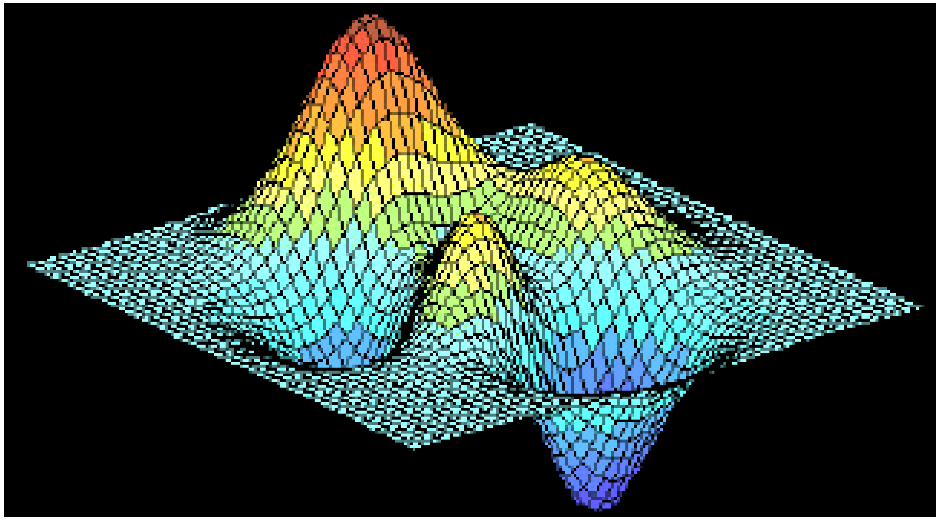
- a method for solving optimization problems based on a natural selection process that mimics biological evolution.

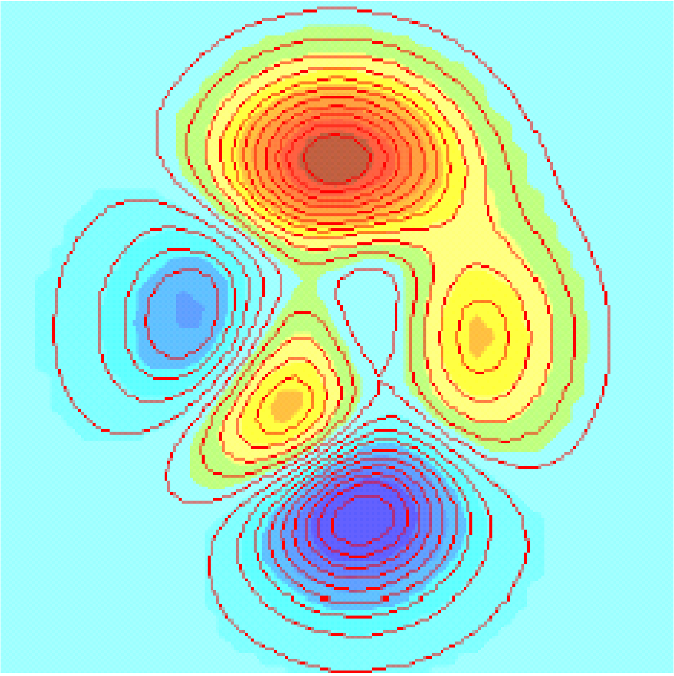
- the algorithm randomly selects individuals from the current population and uses them as “parents” to produce “children” for the next generation.

- over successive generations, the population evolves toward an optimal solution.

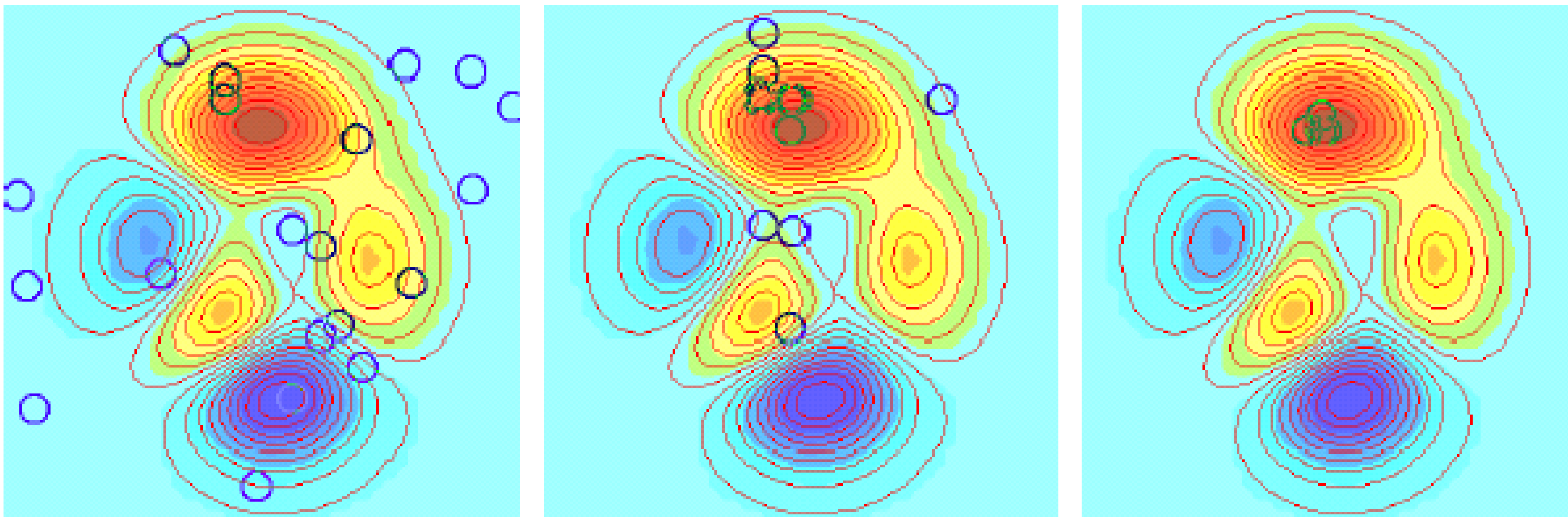
Typical optimization problem using GA

- z = f(x, y) = 3*(1-x)^2*exp(-(x^2) - (y+1)^2) - 10*(x/5 - x^3 - y^5)*exp(-x^2-y^2) -1/3*exp(-(x+1)^2 - y^2).





- GA process:



Initial population　　5th generation　　10th generation

Simplifying analogy

- Chromosome ⟺ Strategy (7-bit strategy)
- Genes ⟺ {R, P, S, etc.}
- Genotype ⟺ (R, P, S, S, P, S, S)
- Phenotype ⟺ {1,0,-1}

GA Mechanism

1. Initialization: generate an initial population at random (i.e. P(i)=P(1) , first generation).

2. Iterate:
   a) evaluate the fitness of individuals in P(i)
   b) select parents from P(i) based on their fitness
   c) generate offspring from the parents using crossover & mutation to form P(i+1)
   d) go back to (a) with P(i+1)

3. Continue until some stopping criteria is satisfied.

<u>Report statistics!</u>

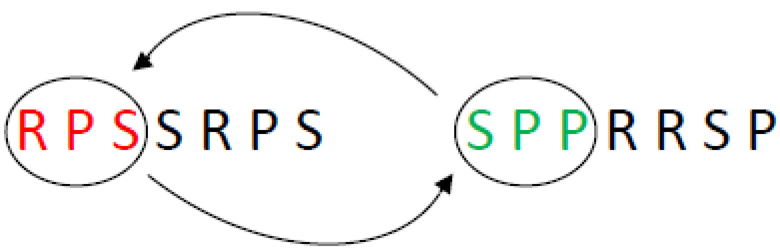
Reproduction Methods

1. Cross-over

1st Generation (parents)

R P S S R P S — Parent 1

S P P R R S P — Parent 2



2nd Generation (offspring)

S P P S R P S

R P S R R S P

2. Mutation

R P S S R P S

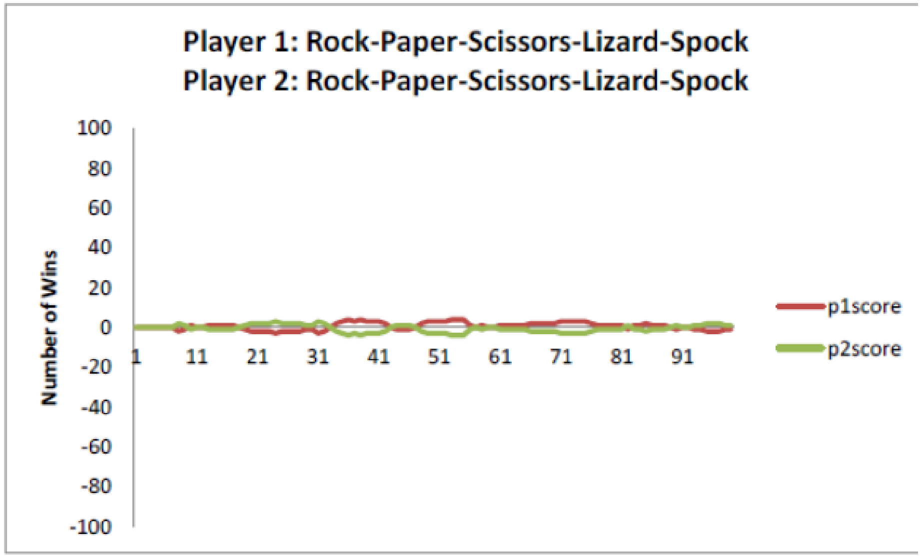
↓

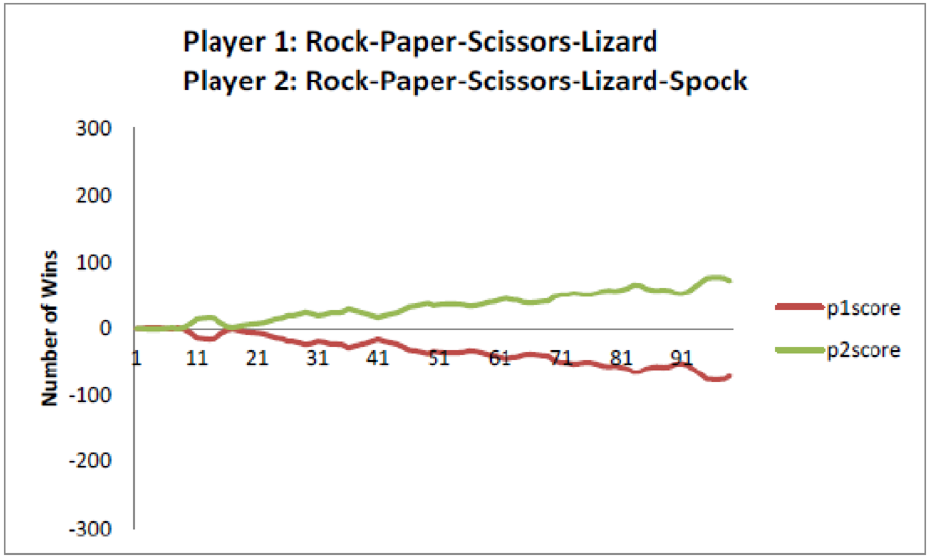
R R S S R P P

Simulation Results:

A. Both are undounded:

B. One is bounded,
the other is unbounded:

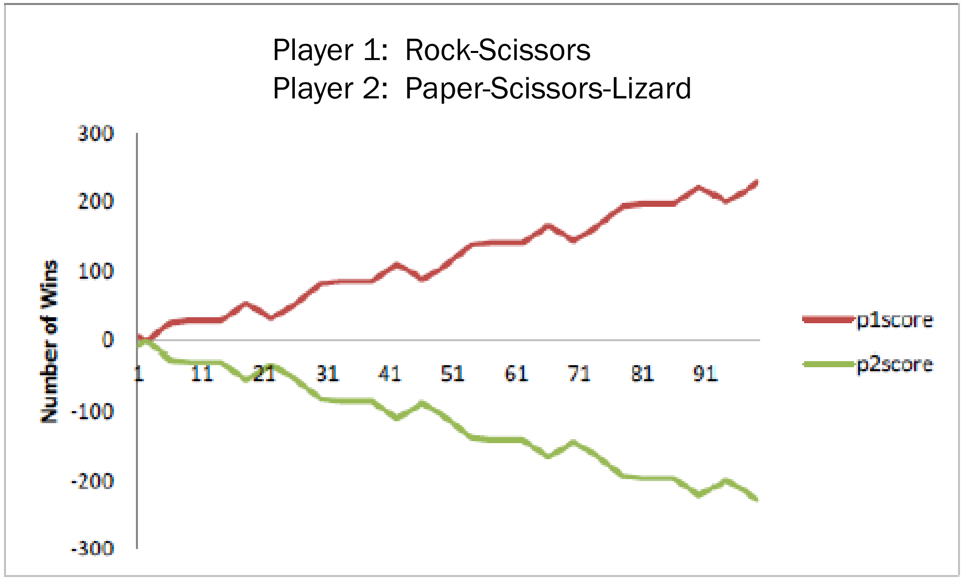




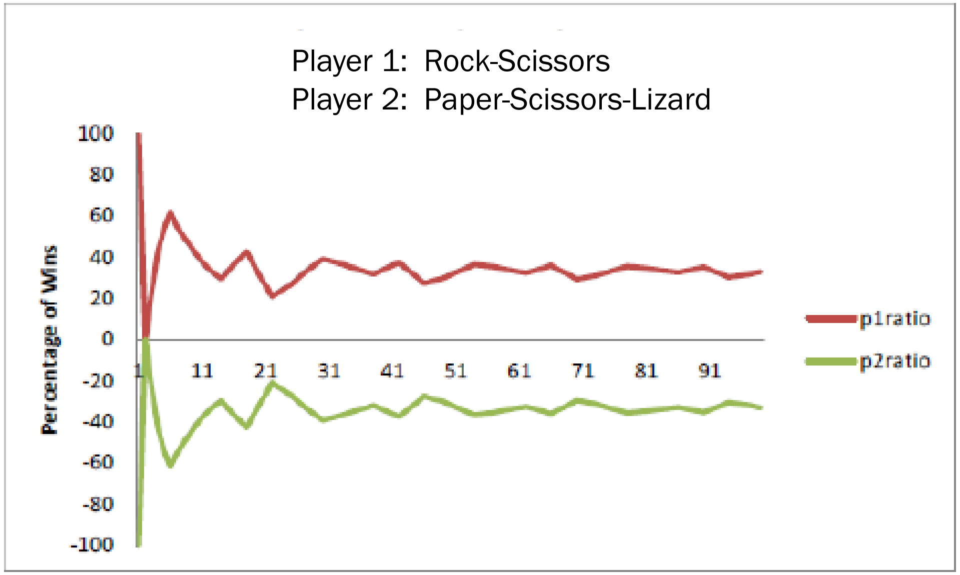
On the average, the result is a draw.

The bounded fellow is almost always defeated.

C. Both are bounded:



Player 1 who has less info is getting higher payoff !



5-option universe

# of options of Player 1	# of options of Player 2	# of possible combinations	# of solutions found
4 opt	5 opt	5	0
3 opt	5 opt	10	0
2 opt	5 opt	10	0
3 opt	4 opt	50	0
2 opt	4 opt	50	0
2 opt	3 opt	100	10

Total # of possible combinations 225

Total # of solutions found 10

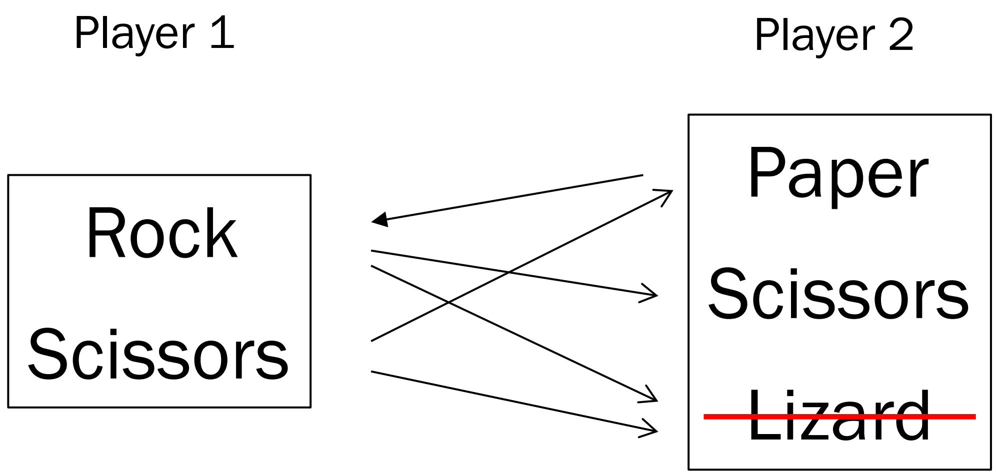
7-option universe

# of options of Player 1	# of options of Player 2	# of possible combinations	# of solutions found
6 opt	7 opt	7	0
5 opt	7 opt	21	0
4 opt	7 opt	35	0
3 opt	7 opt	35	0
2 opt	7 opt	21	0
5 opt	6 opt	147	0
4 opt	6 opt	245	2
3 opt	6 opt	245	1
2 opt	6 opt	147	0
4 opt	5 opt	735	48
3 opt	5 opt	735	4
2 opt	5 opt	441	0
3 opt	4 opt	1225	113
2 opt	4 opt	735	21
2 opt	3 opt	735	98

Total # of possible combinations **5509**

Total # of solutions found **287**

Elimination of weakly dominated strategies:



Player 2

Player 1

	Paper	Scissors	Lizard
Rock	-1, 1	1, -1	1, -1
Scissors	1, -1	0, 0	1, -1

Lemma: In a zero-sum game with a full set of actions Ω for both players, any arbitrary deletion of an action $a_y \in \Omega$ by player y makes a certain action $a_x \in \Omega$ by player x weakly dominated.

Proof:

Let a_y be the action deleted from the roster of actions of player y. Since there are $(n-1)/2$ actions of x that beats a_y, these actions a_x^- will now have negative expected payoffs i.e.

$$a_x^- = \left\{ a_x \in \Omega_x \middle| \sum_{y=1}^{n-1} u_x(a_x, a_y) P_y < 0 \right\}.$$

To show that one of these a_x^- is weakly dominated, pick an action a_x' such that it can beat the same set of y actions that a a_x^- can defeat. There are two of these a_x^- that can fulfill this: One that defeats a_x' and another that is defeated by a_x'. By choosing the latter, we now have an $a_x^* \in a_x^-$ whose $u_x(a_x', \tilde{a}_y) \geq u_x(a_x^*, \tilde{a}_y)$ for all $\tilde{a}_y \in \Omega \backslash a_y$. The action a_x^* is therefore weakly dominated brought about by the removal of action a_y. ■

Proposition 1: A player (fully-rational) who employs all the possible actions in a finite universal set Ω always obtains a higher expected payoff than a player (bounded rational) who only employs a set of actions A, where $A \subset \Omega$.

Proposition 2: For any action set $A \subset \Omega$ that is available to player x, there exists an action set $B \subset \Omega$ available to player y (where the number of elements in B is less than the number of elements in A) such that the expected payoff it gives to y is higher than what x can obtain from employing A.

vs

The fellow with less options wins around 65 percent of the time!

The End

Thank you!