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in the Dynamic Humean Farmer Game**

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# Helping Cost, Assortative Matching and Production Cycles in the Dynamic Humean Farmer Game

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## Abstract

We reformulate the Humean farmer game on the basis of random assignment of advantage and the cost  $e$  of helping in another's harvest. The result is a game that is a coordination game if  $e < \frac{1}{2}$  or a dominant strategy Prisoner's Dilemma Game if  $e > \frac{1}{2}$  which allows a joint treatment of the two interpretations of the Humean farmer game. We employ two behavioral types initially: the conditionally cooperative (H-type) and the free riding (NH-type). We employ replication dynamics with assortative matching and multiple production cycles to investigate which evolutionarily stable (EE) monomorphic population it engenders. We show that the ceiling for effort cost  $e$  to support an EE monomorphic H-type population in the Stag-Hunt game rises to  $(1 + b)/2$  from  $1/2$  in pure random matching case. As the assortative index  $b$  rises, the basin of attraction of the EE H-type solution rises. When the assortative matching is perfect ( $b = 1$ ), in the Stag-Hunt game version ( $0 < e < \frac{1}{2}$ ), the monomorphic NH-type population ( $s^* = 0$ ) is no longer EE while the monomorphic H-type solution ( $s^{**} = 1$ ) is EE; in the Dominant Strategy game version ( $e > \frac{1}{2}$ ),  $s^* = 0$  is EE iff  $e > (3/4)$  while  $s^{**} = 1$  is EE.

Key words: Humean farmer game, helping cost, assortative matching, production cycles, emergence of cooperation

JEL Classification: B0, B15, B31, C73

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## I. Introduction

David Hume (1740) in *A Treatise on Human Nature* wondered how mutually beneficial cooperation could be so prevalent among corn farmers when the nature of the game seems to demand that it should not be because farmers are self-interested (“knaves” in his words) for whom shirking is individually beneficial. The following is his most cited account of the problem:

*“Your corn is ripe today; mine will be so tomorrow. Tis profitable for us both, that I shou'd labour with you to-day, and that you shou'd aid me to-morrow. I have no kindness for you, and know you have as little for me. I will not, therefore, take any pains on your account; and should I labour with you upon my own account, in expectation of a return, I know I shou'd be disappointed, and that I shou'd in vain depend upon your gratitude. Here then I leave you to labour alone: You treat me in the same manner. The seasons change; and both of us lose our harvests for want of mutual confidence and security.”* (Treatise, pp. 520-21)

The situation without additional mechanisms leads to a disaster for the community of “naturally selfish” farmers. But farming communities persist and even prosper. Something is pushing back the disaster. In the modern era, this is reminiscent of the work of Elinor Ostrom and collaborators (1990, 1994, 2000) who documented the avoidance—despite predictions to the contrary by the Olson “zero public good hypothesis”—of the “tragedy of the commons” in small close-knit communities. This problem has now acquired the honorific label of “Farmers’ Dilemma” (Sobel, 1994, Vanderschraaf, 1995) where the pursuit by farmers of individual benefit leads to a socially disastrous outcome. The search for solution to social dilemma games, following David Hume, is really the search for efficiency (“mutual advantage”) through cooperation, in contrast to the search for efficiency through competition following Adam Smith (*The Wealth of Nations*, 1776).

Hume’s own proffered resolution of the problem employs what he calls “convention”: a behavioural rule or contract that is socially beneficial when adhered to by most people. This collective adherence in turn makes the rule self-enforcing in that those who violate only hurt themselves which, in turn, produces the mutually shared expectation that others will, in their turn, adhere (Vanderschraaf, 1998).

*“When this common sense of interest is mutually express'd, and is known to both, it produces a suitable resolution and behaviour. And this may properly enough be call'd a convention or agreement betwixt us, tho' without the interposition of a promise; since the actions of each of us have a reference to those of the other, and are perform'd upon the supposition, that something is to be perform'd on the other part.”* (Treatise, p. 490)

Conventions are supported not only by the “supposition” of performance by the other party but by some penalty implied in the withdrawal of trust:

*“When a man says he promises anything, he in effect expresses a resolution of performing it...and along with that...subjects himself to the penalty of never being trusted again in case of failure.”* (Treatise, page 335)

Hume's fascination with cooperation in dilemma games which include—apart from the Farmer's Dilemma—the institution of private property and the digging of ditches and especially his resort to conventions as possible solution has attracted extensive scholarship in the modern era (Vanderschraaf, 1998). Conventions have been widely interpreted in the modern era as equilibria of strategic games. Here however there is divergence as to which games: Lewis (1969) viewed conventions as equilibria of coordination games whereas Sobel (1994) models the Farmers' Dilemma as a static Prisoner's Dilemma game (PDG). Vanderschraaf (1995, 1998) prefers the extensive form representation of the Prisoner's Dilemma as faithful to Hume's sequential farmer interaction. As an example of a Humean convention, Vanderschraaf (1998) identifies the trigger strategy equilibria of repeated PDGs. The interpretation of the Humean Farmer Game as a coordination game suffers from the drawback that in pure coordination games, whichever equilibrium one starts from, unilateral deviation is immediately punished; whereas Hume's cooperative status quo seemed to be a profile from which unilateral deviation is beneficial to the deviant. Lewis's interpretation is however supported by many familiar modern conventions such as "right-hand side driving" or the use of money where the convention is self-enforcing. If the game is a coordination game, the problem boils down to the transition from the bad Nash to the superior Nash equilibrium. If the game is a PDG, the problem is not only one of transition to a mutually beneficial solution but also stability after attainment.

The condition "mutually beneficial" implies that the coordination game in question is in addition a Stag-Hunt game where the convention-favored Nash equilibrium payoff dominates the other. The problem here is now known as equilibrium selection (see e.g., Harsanyi and Selten, 1988; Kandori, Mailath and Rob, 1993; Young, 1998; Skyrms and Pemantle, 2000). As observed, once attained, the payoff dominant Nash equilibrium is self-enforcing and so requires no other mechanism for stability, which is not true of the Prisoner's Dilemma Game.

We revisit the Humean farmer game taking on board three salient features of the Humean paradigm: (1) the coexistence of two types of farmers, viz., the reciprocal or conditionally cooperative type and the free riding type, both of which are encompassed by Hume's rationality; (2) the reckoning of the payoffs according to a random rather than a fixed assignment of advantage in each production cycle, (3) an explicit modelling of the helping cost  $e$ . Further on we extend the model to include (4) the penalty meted upon revealed shirkers by way of assortative matching; and finally (5) the shadow of the past in the emergence of cooperative farming communities implied in Hume's phrase "slow progression".

*"Nor is the rule concerning the stability of possession the less deriv'd from human conventions, that it arises gradually, and acquires force by a slow progression, and by our repeated experience of the inconveniences of transgressing it. On the contrary, this experience assures us still more, that the sense of interest has become common to our fellows, and gives us a confidence of the future regularity of their conduct."* (Treatise, p. 490)

Vanderschraaf (1998) opines, and we agree, that "...Hume's qualitative description conveys the basic idea that equilibrium can emerge as the result of some dynamical adjustment process." This dynamical process over generations we interpret with Vanderschraaf as an evolutionary process employing the replication dynamics. But even here the attainment and stability of cooperation is

contested. The Agents learn by direct experience to detect and avoid revealed shirkers. Theirs has become the favorite pathway to stable cooperation in the last decade and a half (e.g., Greiff, 2013). The message from the evolutionary replicator dynamic treatment of the dilemma games is in fact a mixed bag for the cooperative solution: If the game is prisoner’s dilemma variety, pure random matching does not allow a stable cooperative stationary state (Bergstrom, 2002). If the game is a coordination game of the Stag-Hunt variety, both the cooperative and free riding stationary states are stable but Kandori, Mailath and Rob (1993) and Young (1998) have shown that the risk dominant (shirking) solution will outlast the cooperative solution over the long run if random mutation is allowed and matching is purely random. Skyrms and Pemantle (2000) and Pemantle and Skyrms (2004) have thus dropped pure random matching in favor of re-enforcement learning to render the cooperative stationary state of the coordination game resistant to invasion and thus persistent in the long run. Agents learn about agent past behavior and deploy this to avoid shirkers in subsequent periods. This has become the favored pathway to mutation- resistant monomorphic cooperative communities.

In Section II, we reconstruct the Humean Farmer Game according to the salient features above. We show that the Humean farmer game has a dual character: it is a coordination game of the Stag Hunt variety when helping cost  $e < \frac{1}{2}$ ; it becomes a dominant strategy PDG if  $e > \frac{1}{2}$ . In Section III, we subject the Humean Farmer Game to a Malthusian replication dynamics where farmers learn about others and employ a purposive assortative matching. We give the conditions for a population of all cooperators to be evolutionary equilibrium (EE) and for both pure cooperators and pure non-cooperators to be EE. These conditions are interplays of the sizes of  $b$  and  $e$ .

## II. The Humean Farmer Game: Random Assignment of Advantage

David Hume’s Farmers’ Dilemma Game is a strategic interaction game where farmer  $F_2$ ’s corn ripens first and farmer  $F_1$  decides to either offer help  $H$  or no help  $NH$ . Thus, the action set of farmer  $i, i = 1, 2$ , is  $(H, NH)$ . In Hume’s rendition, if  $F_1$  does offer help,  $F_2$  is “in possession of advantage” because he can refuse help when  $F_1$ ’s corn ripens. If both farmers are opportunistic (“knaves” sic Hume) and know themselves to be such, neither will offer the other help and the disaster outcome will prevail. We make the following simple assumptions on the production technology  $f(\cdot)$  and cost:  $f(H, H) = 1$  and  $f(H, NH) = f(NH, H) = 0$ . The latter reflects Hume’s “loss of harvest.” We assume that the cost of helping effort is  $e$ :  $0 < e < f(H, H) = 1$ . For contrast, we present the Humean game among free riding farmers with known position of advantage.

### A. Single-Type Humean Farmer Game: Fixed Assignment of Advantage

Let both farmers be free riding. The static symmetric Humean farmer game in normal form is given in **Table 1**:

**Table 1.** Static Humean Farmer Game

		$F_2$	
		$H$	$NH$
$F_1$	$H$	$(1 - e), (1 - e)$	$-e, 1$
	$NH$	$1, -e$	$0, 0$

Note that  $(1 - e)$  is the payoff of each farmer if both offer help: 1 being the output accruing to  $F_1$  and  $-e$  is the effort cost to  $F_1$  in helping  $F_2$  and vice-versa. If  $F_1$  helps but  $F_2$  reneges,  $F_1$  gets zero output but incurs  $-e$  for helping  $F_2$ . This is a dominant strategy game PDG with (NH, NH) as the dominant strategy equilibrium similar to Sobel's (1994). One may prefer the extensive form of the game (Vanderschraaf (1998); it is easy to see that (NH, NH) is the unique sub-game perfect equilibrium of the extensive form rendition of the game whoever makes the first move.

## B. Two-Type Humean Farmer Game with Random Assignment of Advantage

We assume with Hume that the “conditionally cooperative type” (those with a tendency to recognize and pursue “reciprocal advantage”) co-exists in the initial farmer population together with purely selfish farmers (“knaves”). For the conditionally cooperative type we use the short-hand H-type—those farmers who extend help to a partner when the partner’s corn ripens today, and refuse help tomorrow only if his own ripens today and the partner refuses help today. We label the second as NH-type – those farmers who refuse help to a partner today and also refuses help tomorrow even if the partner helped him today. That is, NH-types are the knaves or shirkers. The use of two types in the initial population is common in Biology (see e.g., Maynard Smith, 1964 and Cohen and Eshel, 1976). The type of one’s partner is not directly observable but is known only up to a probability distribution. Let  $s$  be the probability that one’s partner is H-type;  $(1-s)$  that the partner is NH-type. Let whose harvest ripens first be a random event with probability  $(\frac{1}{2})$ .

The payoff matrix is shown in **Table 2**.

**Table 2.** Payoff Table: Two-Type Humean Farmer Game with Random Assignment of Possession of Advantage

		$F_2$	
		$H$	$NH$
$F_1$	$H$	$(1 - e), (1 - e)$	$-e/2, 1/2$
	$NH$	$1/2, -e/2$	$0, 0$

Suppose  $F_1$  who is an  $H$ -type partner with  $F_2$  who is another  $H$ -type, the average payoff is  $(1 - e)$ : that is, if  $F_1$ 's corn ripens today, he realizes output 1 with the help of  $F_2$  who incurs helping cost  $(-e)$ ; if  $F_1$ 's corn ripens tomorrow, he will still realize output 1 with help of  $F_2$  but will have incurred cost  $(-e)$  helping  $F_2$  whose corn ripened today. Thus,  $(1 - e)$  for  $(H, H)$ . When  $F_1$  who is  $H$ -type partners with  $F_2$  who is  $NH$  type, the expected payoff is  $(-e/2)$ : that is, if  $F_1$ 's corn ripens today with probability  $(1/2)$ ,  $F_1$  will realize zero output without help from  $F_2$ , while if  $F_1$ 's corn ripens tomorrow with probability  $(1/2)$ ,  $F_1$  will also realize zero output but will have helped  $F_2$  today thus incurring helping cost  $(-e)$ ; thus the expected  $(-e/2)$ .

Suppose  $F_1$  who is  $NH$ -type partners with  $F_2$  who is  $H$ -type, the average payoff is  $(1/2)$ : that is, if  $F_1$ 's corn ripens today with probability  $(1/2)$ , he realizes output 1 with help from  $F_2$ ; if  $F_1$ 's corn ripens

tomorrow (the flip side being  $F_2$ 's corn ripens today),  $F_1$  will get no help from  $F_2$  (since  $F_2$  did not get help from  $F_1$  yesterday) and will realize zero output. Thus the average payoff is  $(1/2)$ .

Suppose both  $F_1$  and  $F_2$  are  $NH$ -type, the average payoff is 0. Regardless of whose corn ripens today, each will realize zero output and the average payoff is 0.

### C. Two Versions of the Humean Farmer Game

Depending on the size of the helping cost  $e$  we have two versions:

*Version 1:*  $0 < e < (1/2)$ . The game becomes a Coordination Game (CG) of the Stag Hunt variety with one Nash equilibrium  $(H, H)$  strictly payoff dominating the other equilibrium  $(NH, NH)$ . Helping effort cost being low ( $e < (1/2)$ ) can be understood as the case when resources are scarce, going it alone has poor prospect and thus the opportunity cost of helping is low. Heinsohn and Packer (1995) give evidence that among lion prides, when resources are scarce, pride members cooperate more readily to hunt. It is well-established that under purely random matching, the replication dynamic (RD) treatment the Stag-Hunt Game has three stationary equilibria, all- $H$  types ( $s^* = 1$ ), all- $NH$  types ( $s^* = 0$ ) and a co-existing mixed population ( $0 < s^* < 1$ ). Only the first two are evolutionary equilibria (evolutionarily stable). Which of these two will prevail in the long-run depends upon the initial fraction  $s$  of  $H$ -types in the population. Thus, Lewis' (1969) rendition of the Humean Farmer's Dilemma as a coordination game of the Stag-Hunt variety finds corroboration here. Since  $(H, H)$  is self-enforcing, the convention is simple: always choose  $H$  and the other will choose  $H$ . As observed, the problem of equilibrium selection arises.

*Version 2:*  $(1/2) < e \leq 1$ : The Humean farmer game is now a PDG of the Dominant Strategy variety with  $(NH, NH)$  as dominant strategy equilibrium. Helping effort cost being high ( $e > (1/2)$ ) can be understood as the case when resources are abundant and thus the opportunity cost of helping the other is high. Cooperation becomes less attractive (as exemplified again by lion pride behavior in Heinsohn and Packer, 1995). Sobel's (1994) and Vanderschraaf's (1998) interpretation of the Humean Farmer Game requires  $e$  to be high in this case. This by the way is also the common vehicle for most research on the emergence of altruism (see, e.g., Bergstrom, 2002)

These two versions have different footprints under replication dynamics. It is well-known that the RD treatment of the Dominant Strategy Game with pure random matching results in only one evolutionary equilibrium, the all- $NH$  population (Bergstrom, 2002). Thus, the all- $H$  equilibrium (also the cooperative equilibrium) will not prevail in the long-run however large the initial fraction of  $H$ -types is. The Stag Hunt Game in contrast will under replication dynamics allow both an All- $NH$  and an All- $H$  population to be evolutionary equilibria.

The Stag-Hunt Game is at the heart of the equilibrium selection debate (Harsanyi and Selten, 1988). Kandori, Mailath and Rob (1993) and Young (1998) have shown that employing purely random matching but introducing random mutation will eventually destabilize the cooperative equilibrium. Ellison (1993) showed further that adding local interaction alone under the same random matching and random mutation is not sufficient to save the cooperative equilibrium. The prospect of explaining the

emergence of the all- $H$  population among corn farmers by evolutionary hardwiring seems closed in evolutionary game theory with a pure random matching (Skyrms, 2004, 2007).

We interpret Hume's phrase "...that the sense of interest has become common to our fellows..." as the engendering of the all- $H$  equilibrium under replication dynamics in either of the two versions of the Humean two-cycle-two-type game. We know from Bergstrom (2002) that random matching seems a dead end for the purpose. We therefore drop it in favor of non-random matching. We tackle this next.

## II. Learning and Assortative Matching

In lieu of purely random matching with fixed interaction structure, we follow the purposive matching literature in employing mechanisms that allow  $H$ -types to match with  $H$ -types with increasing probability. As observed, Skyrms and Pemantle (2000), Pemantle and Skyrms (2004) and Skyrms (2004) employed re-enforcement learning and networking in small groups to guarantee the persistence of the cooperative equilibrium.

This section employs a simplest learning and matching routine: assortative matching (also referred to a 'routine learning' see, e.g., Montet and Serra, 2003). It is the simplest learning routine borrowed from Psychology (Roth and Erev, 1995; Bush and Mosteller, 1955). This has been employed in Biology (L.L. Cavali-Sforza and Feldman, 1981; Cohen and Eshel, 1976). To allow for learning, we also imbed multiple production cycles in one lifetime in the dynamic Humean farmer game. We start with a simple two-production cycles in a lifetime. Then we extend the model to  $n$ -cycles.

### Learning in the Two-Cycle Lifetime

Let  $s$  be the initial fraction of  $H$ -type in the population. In cycle 1, farmers do purely random matching, that is,  $s$  is the probability of matching with an  $H$ ,  $(1 - s)$  the probability of matching with an  $NH$ . In cycle 1, if the agent reveals himself as  $H$ , the assortative matching probability of meeting another  $H$  in cycle 2 is:

$$P(H, 2) = s + b(1 - s), 0 \leq b \leq 1.$$

If the agent reveals himself to be  $NH$  in cycle 1, the probability of meeting and  $H$  in cycle 2 is

$$P(NH, 2) = s(1 - b).$$

' $b$ ' is the fixed assortative matching index. The size of ' $b$ ' reflects inversely the size of the group or the extent of local interaction in question. One may if so desired render  $b$  endogenous through the Skyrms and Pemantle (2000) process. By contrast, we just adopt a learning and matching routine without worrying about how it arises. If  $b = 0$ , we have the case of pure random matching or fixed interaction structure; if  $b = 1$ , we have perfect correlation matching which is more likely the smaller is the interacting local group.



Given the payoffs in **Table 2**, the expected fitness of an *H*-type is:

$$EU_H = \{s(1 - e) + (1 - s)(-e/2)\} + \{[s + b(1 - s)](1 - e) + [1 - [s + b(1 - s)]]( -e/2)\}$$

where the first parenthesized expression is the cycle 1 payoff and the second parenthesized expression is that of cycle 2. This simplifies into:

$$EU_H = 2s(1 - e) - (e/2)(1 - s)(2 - b) + b(1 - s)(1 - e).$$

That of *NH* is:

$$EU_{NH} = \{(s/2) + (1 - s)(0)\} + \{s(1 - b)(1/2)\} = (s/2) + s(1 - b)/2 = (s/2)(2 - b).$$

The Malthusian replication dynamics is given by:

$$(ds/dt) = s[EU_H - (sEU_H + (1 - s)EU_{NH})] = s(1 - s)[EU_H - EU_{NH}]$$

$$(ds/dt) = s(1 - s)M$$

where

$$M(s) = [2s(1 - e) - (e/2)(1 - s)(2 - b) + b(1 - s)(1 - e) - (s/2)(2 - b)].$$

Stationary solutions exist at  $s^* = 0$ ,  $s^{**} = 1$  and at  $s^{***}$ ,  $0 < s^{***} < 1$ , when  $0 < s < 1$  but  $M(s) = 0$ . Suppose for the moment that  $b = 0$  (pure random matching),  $M(s) = 0$  gives  $s^{***} = \{(e/2)(2 - b)/(1 - e)(2 - b)/2\} = e/(1 - e) < 1$  since  $e < 1$ . With  $b = 0$ , there is no stationary solution between zero and one when the game is dominant strategy PDG ( $e > 1/2$ ) since  $s^{***} = e/(1 - e) > 1$  is disallowed. So there can be only one stationary solution which happens to be  $s^* = 0$ . In this case of the Stag-Hunt version, with  $b = 0$ , both  $s^* = 0$  and  $s^{**} = 1$  are stable so  $s^{***} = e/(1 - e)$  is unstable. These are standard results (both the Maynard Smith's Haystack model and the Cohen-Eshel model which display two stable equilibria, one of them cooperative under random matching are Stag-Hunt Games—see Bergstrom, 2002). Thus, adding production cycles by itself has no effect on the emergence of the cooperative equilibrium under random matching. These standard results now serve as our benchmarks. How do these results change if, as Hume surmised, farmers learn and resort to avoiding free riders modeled here by assortative matching ( $b > 0$ )?

We return to  $b > 0$ . The basin of attraction of  $s^{**}$  is  $(1 - s^{***})$ .

The derivative of  $(ds/dt)$  with respect to  $s$  is

$$[d(ds/dt)/ds] = (1 - s)M - sM + s(1 - s)[dM/ds].$$

At  $s = 0$ , this reduces to:

$$[b[1 + (1/2) - e] - e].$$

At  $s = 1$ , this reduces to:

$$[(1 - b) - 2(1 - e)].$$

We are interested in the cases when  $s = 0$  and/or  $s = 1$  are stable stationary solutions, that is, are evolutionary equilibria (EE). We know that (see e.g., Montet and Serra, 2003):  $s = 0$  is an evolutionary equilibrium iff  $[b[1 + (1/2) - e] - e] < 0$ .

This translates into  $2e > [3b/(1 + b)]$ . On the other hand,  $s = 1$  is evolutionary equilibrium iff  $[(1 - b) - 2(1 - e)] < 0$ . This translates into  $(1 + b) > 2e$ . The interaction between the assortative index  $b$  and helping cost  $e$  in supporting EE solutions is given formally in the following:

*Proposition 1:* In the dynamic Humean farmer game: (i)  $s^* = 0$  is an evolutionary equilibrium iff  $2e > [3b/(1 + b)]$ ; (ii)  $s^{**} = 1$  is an evolutionary equilibrium iff  $(1 + b) > 2e$ . (iii) both  $s^* = 0$  and  $s^{**} = 1$  are EE if  $(1 + b) > 2e > [3b/(1 + b)]$ .

For further clarification on the joint role of  $b$  and  $e$  in supporting EE in the dynamic Humean Farmer game, the following follows immediately from *Proposition 1*:

*Corollary:* (i) For any  $0 < b \leq 1$ ,  $s^{**} = 1$  is EE of the dynamic Humean Farmer game iff  $e < (1 + b)/2$ , (ii) For any  $0 < b \leq 1$ ,  $s^* = 0$  is EE of the dynamic Humean Farmer game iff  $e > 3b/2(1 + b)$ .

For example, if  $b = (1/2)$ ,  $e < (3/4)$  ensures that  $s^{**} = 1$  is EE while  $e > (1/2)$  ensures that  $s^* = 0$  is EE; finally both  $s^{**} = 1$  and  $s^* = 0$  are EE if  $(1/2) < e < (3/4)$ . By contrast, if  $b = 0$  (pure random matching),  $1/2$  is the allowable ceiling for  $e$  which will support a stable  $s^{**} = 1$ . Any  $b > 0$  raises this ceiling for  $e$ . This is corroborated by *Proposition 2* below on the basin of attraction of  $s^{**} = 1$ .

*Proposition 2:* For any effort cost  $e < (1/2)$  (Stag Hunt Game), a rise in “ $b$ ” raises the basin of attraction of  $s^{**} = 1$ .

*Proof:* Totally differentiating  $M(s)$  and solving for  $(ds/db)$ , we get  $(ds^{**}/db) = (1 - e)(4e - 4)/[2(1 - e) - b(1 - e)]^2 < 0$  for  $e < 1$ . The basin of attraction of  $s^{**} = 1$  is  $[1 - s^{***}]$ . This value increases monotonically as  $b$  rises.

Thus, the effect of introducing assortative matching (moving from  $b = 0$  some  $b > 0$ ) is to increase the basin of attraction of the cooperative equilibrium in the Stag Hunt version.

To put the result in sharper perspective, we consider the corner case of perfect assortative matching ( $b = 1$ ) to contrast with its polar opposite the perfectly random matching ( $b = 0$ ). We have:

*Proposition 3:* Suppose there is perfect assortative matching ( $b = 1$ ). Then: (i) In the Stag-Hunt game version ( $0 < e < 1/2$ ), (i.a)  $s^* = 0$  is not EE, (i.b)  $s^{**} = 1$  is EE. (ii) In the Dominant Strategy game version ( $e > 1/2$ ), (ii.a)  $s^* = 0$  is EE for  $e > (3/4)$ ; (ii.b)  $s^{**} = 1$  is EE.

To see this, let  $b = 1$ . The condition  $(1 + b) > 2e$  for  $s^{**} = 1$  as EE becomes  $2 > 2e$  which is always true for  $e < 1$ ; likewise, the condition for  $s^* = 0$  as EE becomes  $2e > (3/2)$  which is true only for  $e > (3/4)$ . Thus,  $s^* = 0$  has ceased to be EE in the Stag-Hunt version. This is also clear from the fact that for  $b = 1$ , the unstable stationary solution  $s^{***} = (e - 2)/(1 - e) < 0$  for  $e < 1$  so that the basin of attraction of  $s^{**} = 1$  is effectively 1. However,  $s^* = 0$  remains EE for the Dominant Strategy version if  $e > (3/4)$ . This makes sense in view of the previous observation on lion pride cooperation (Heinsohn and Packer, 1995):  $e$  is the opportunity cost of helping; if this is very high as when ambient resources are abundant, the non-cooperative population or, in the phrase usually encountered in this area (e.g., Skyrms and Pemantle, 2004), a purely non-cooperative “hare-hunting” population can persist.

Meanwhile,  $s^{**} = 1$  is EE in either version of the dynamic Humean Farmer Game. Perfect assortative matching has rendered the pure H-type equilibrium the likelier outcome of the replication dynamics of the Humean Farmer Game. The NH or shirking equilibrium ceases to be stable in the Stag Hunt version and barely survives in the Dominant Strategy version of the Humean game.

With full accounting for the Humean Farmer Game features (two types of farmers: reciprocal or free riding, random assignment of advantage and assortative matching), cooperative communities do emerge as a long-term equilibrium in either version. These results are robust against the introduction of discontinuous random mutation because free riding mutants introduced among the cooperative population will not prosper as they become identified and isolated. The drawback in these results is their dependence on the size of the assortative index  $b$  which will conceivably be small with a large population. A small  $b$  will shift the burden of supporting the cooperative equilibrium to the cost of helping  $e$  (*Corollary*). In the next section we explore how a higher frequency of interaction may remedy this shortcoming for larger population.

#### IV. The Case of More than Two Production Cycles

Adding a second production cycle allows learning and assortative matching to kick in and generate very different dynamic long-run results. In this section, we generalize to  $n > 2$  production cycles. This is akin to the length of time that a population in the haystack model of Maynard Smith (1964) stays in isolation which, if sufficiently long, allows a cooperative equilibrium to be stable in the Cohen-Eshel model (see also Bergstrom, 2002).

We saw in the last section that the probability of an  $H$ -type matching with another  $H$ -type rises from  $s$  in cycle 1 to  $[s + b(1 - s)]$  in cycle 2, while an  $H$ 's probability of matching with an  $NH$  decreases from  $(1 - s)$  to  $\{1 - [s + b(1 - s)]\}$ . The progression as the number of cycles  $n$  rises beyond 2 follows the same pattern.

Let  $P(j)$  be the probability of an  $H$ -type meeting an  $H$ -type in cycle  $j$ . The progression of the probability through  $n$  is as follows:

$$P(2) = s + (1 - s)b = s + (1 - s)b$$

$$P(3) = s + (1 - s)b\{(2 - b)(1 - b) + 1\} = s + (1 - s)bB_2$$

where

$$B_2 = \{(2 - b)(1 - b) + 1\}.$$

$$P(4) = s + (1 - s)b\{[(2 - b)(1 - b) + 1](1 - b) + 1\} = s + (1 - s)bB_3$$

where

$$B_3 = \{B_2(1 - b) + 1\}.$$

$$P(5) = s + (1 - s)b\{[(2 - b)(1 - b) + 1](1 - b) + 1\}(1 - b) + 1\} = s + (1 - s)bB_4$$

where

$$B_4 = \{B_3(1 - b) + 1\}.$$

$$P(6) = s + (1 - s)b\{[(2 - b)(1 - b) + 1](1 - b) + 1\}(1 - b) + 1\}(1 - b) + 1\} = s + (1 - s)bB_5$$

where

$$B_5 = \{B_4(1 - b) + 1\}.$$

$$P(n - 1) = s + (1 - s)bB_{n-2}$$

where

$$B_{n-2} = \{B_{n-3}(1 - b) + 1\}.$$

$$P(n) = s + (1 - s)nB_{n-1}$$

where

$$B_{n-1} = \{B_{n-2}(1 - b) + 1\}.$$

It is easy to see that for any  $b > 0$ : (1)  $bB_1 > bB_2 > bB_3 > \dots > bB_{n-1} > bB_n$  and (2)  $bB_n \rightarrow 1$  as  $n$  becomes large. For example, if  $b = (1/2)$ ,  $bB_1 = (7/8)$ ,  $bB_3 = (15/16)$  and  $bB_4 = (31/34)$ , etc. Thus,  $P(j) < P(j + 1) < P(j + 2)$ ... The corresponding probability for an  $H$  to match with a  $NH[1 - P(j)]$  decreases accordingly. This monotone progression is more direct than the progression of probabilities in reinforcement learning where the inputs to the learning are units of positive or negative experiences (see, e.g., Greiff, 2013).

What this shows is that any assortative matching index ( $b > 0$ ), however modest, can seed a monotonically rising correlation of beneficial matches if more and more production cycles to each lifetime are allowed. For large enough number of cycles, the matching correlation approaches 1 and virtually ensures the stability of the cooperative equilibrium.

## V. Summary:

D. Hume's corn farmer game is one of the earliest and arguably among the most influential early example of a social dilemma game where the pursuit of individual well-being leads to inferior social outcomes. Cooperation among self-interested agents ('knaves' in Hume's terminology or 'Foole' in Hobbes, 1651) is a universal desideratum that does not always get attained. Hume's own explanation for the persistence of cooperation among corn farmers despite the obvious short-term personal benefit for shirking involved the institution of 'convention' to which most of the members adhere as it were automatically, and which has become imbedded among members of these communities through repeated—perhaps generational—interaction. Other names for these are 'trust', 'norms', 'social capital', 'social contract', or 'heuristics' that conduce towards cooperation without conscious forethought: in Hume's phrase, "without any real kindness." The emergence of cooperation among self-interested agents has been a challenge to biology and the social sciences. Most attempts at a resolution involve the use of game theory—specifically evolutionary game theory—where replication dynamics has been the analytical weapon of choice. In this endeavor, the debate has become distilled to mean the emergence and stability of a monomorphic population of cooperators.

In this paper, we first attempt the reconstruction of the Humean corn farmer game utilizing three principal components extant in Hume—the random assignment of advantage, the presence of two types of farmers: conditionally cooperative and free riding types in the initial population, and the capacity to learn and punish shirkers through avoidance. We focus on the role of the cost of helping ('loss of fitness' in the evolutionary biology). The size of the helping cost delineates two types of Humean farmer game: the Stag-Hunt Game version and the Dominant Strategy Game version. Only the Dominant Strategy version carries the Sobel label 'Farmers' Dilemma'. Thus accounting for the Humean farmer game features enlarges the compass of the Humean game.

Subjecting the Dominant Strategy PDG version to replication dynamics under purely random matching does not allow the monomorphic cooperative equilibrium to emerge in keeping with the literature (see, Bergstrom, 2002). The cooperative equilibrium emerges as one of the two stable equilibria of the dynamic Stag Hunt version. But allowing for random mutation undermines the long-term viability of the cooperative equilibrium.

In lieu of pure random matching, we adopt assortative matching where conditionally cooperative types learn to increasingly interact with other conditionally cooperative types and to avoid free riding types. Assortative matching serves as a kind of punishment for free-riders. To do this we allow multiple production cycles in one lifetime. For two cycles, we give the condition for the emergence of a stable monomorphic H-type equilibrium in the Dominant Strategy PDG version and the Stag-Hunt version. These conditions depend on the relative sizes of the helping cost and the assortative index. In the case of perfect assortative matching more appropriate perhaps for small group interaction or for many production cycles in one lifetime, the stable monomorphic NH-type equilibrium is still possible only for large helping cost but the stable monomorphic H-type population is now as it were the rule.

We show that the ceiling for effort cost  $e$  to support a stable monomorphic H-type population rises to  $(1 + b)/2$  from  $1/2$  in pure random matching case. As the assortative index  $b$  rises, the basin of attraction for the stable monomorphic H-type solution rises. When the assortative matching is perfect ( $b = 1$ ), in the Stag-Hunt game version ( $0 < e < 1/2$ ),  $s^* = 0$  (the monomorphic NH-type population) is no longer EE while  $s^{**} = 1$  (the monomorphic H-type solution) is EE; in the Dominant Strategy game version ( $e > 1/2$ ),  $s^* = 0$  is EE iff  $e > (3/4)$  while  $s^{**} = 1$  is EE.

We then extend the model to  $n$  cycles and show that for any positive value of the assortative index, however small, the probability an  $H$ -type matching with another  $H$ -type rises monotonically with the rise in the number of cycles. Thus, the perfect assortative matching case above is just another way of saying that the number  $n$  of production cycles is large. The foregoing adds a new twist to the idea of economic development as time compression, that is, development as utilizing less and less time for each activity, thus, allowing more and more cycles of interaction in a given period of time. Among others, as is clear from the results here, this acts to raise the likelihood of beneficial matches, to isolate misfits and thus to attain higher productivity!

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