RISK SHARING AND LAYOFF RISK IN PROFIT SHARING

by

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Abstract

We show that if the employer is risk averse, however slightly, there is always a profit sharing contract that will Pareto-dominate the spot wage contract in the sense of pure risk sharing. The smaller is employer risk aversion, the narrower is the room for profit sharing. The higher the workers value employment stability (less layoff risk), the more Pareto attractive is profit sharing regardless of employer risk aversion.

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Introduction

Profit sharing is an idea mired in controversy. The storm whipped up by M. Weitzman’s (1984) proposal to alter the macroeconomic landscape by subsidizing the shift to profit sharing tends to color every consideration (Wadhwani, 1987; Blanchflower and Oswald, 1988; Rees, 1991). For example, the issue of whether demand for labor will rise with a shift to profit sharing is practically ubiquitous. While the controversy has led to profound insights into many aspects of profit sharing (see, e.g., Estrin, S.; Grout, P. and Wadhwani, S., 1987) a good deal of them being rather disparaging vis-a-vis the Weitzman thesis, it has also led to neglect of some aspects. Meanwhile, profit sharing continues to survive and even gain ground among emerging nimble organizations, e.g., U.S. steelmaker Worthington Industries. Without government intervention, private organizations are beginning to adopt variations of profit-sharing or profit-related pay. There must somehow be organizational value-added associated by profit-related pay that may or may not be related to macroeconomic outcomes. This paper attempts to shed light on such neglected dimension.

Profit sharing encompasses several important issues (Pohjola, 1992): worker motivation, production efficiency, risk sharing and distribution of value-added. This paper will deal primarily with risk sharing; more specifically with whether and when it adds to the attraction of profit sharing. In particular, it attempts to bring out the relation between employer and worker risk attitude and the distribution of value-added in a welfare improving risk sharing under profit sharing. It is of interest that despite the voluminous literature on profit sharing, the risk-sharing angle seems ignored. M. Hellwig (1987) observes as much in discussion of profit sharing.

The earliest attempt at unlocking the risk sharing angle of profit sharing may have been Aoki (1979, 1984). He showed that if employers are more risk-averse than the employees (both with constant absolute risk aversion), there exists a linear sharing rule involving a guaranteed pay and a random sales revenue related bonus which is Pareto optimal and dominates the straight wage contract. Aoki’s firm is sales revenue maximizing rather than profit maximizing and employment is fixed. The unpleasant aspect of this result is that the usual assumption in risk-sharing models is that employers are less risk-averse than workers (Stiglitz (1974) on sharecropping; Azariadis (1975) on implicit contracts, Fabella (1991) on trader-farmer linkage). This is natural since employers choose to be risk takers and have better access to capital markets and other risk diversifying instruments. Profit sharing shifts some risk from employers who are willing and able to bear it to workers who are less so. In the words of Blinder (1992), "that does not seem a good idea." Aoki’s result argues in favor of why sharing is less widespread than it should. Pohjola (1990) considers a utilitarian monopoly union that dictates the
terms of the pay contract and appropriates all the benefits in excess of spot con profit. He finds among others that profit sharing still makes sense even if workers are risk neutral as long as information is symmetric and production socially profitable. This has the same drawback as Aoki's. Furthermore, if workers are risk-averse, the basic wage (the nonrandom component of the payment package) exceeds the alternative (spot) wage. If firms had some say at all, they will naturally balk at the idea of sharing their profit and also paying guaranteed higher than spot wage.

This paper will revisit the risk sharing angle of profit sharing to determine (a) whether and (b) how the extent of worker's risk aversion being greater than employer risk aversion still allows risk sharing to be a positive feature of profit sharing. Like Aoki and in contrast to Pohjola, the employer takes the initiative.

II. The Model

Consider a firm with revenue function \( \hat{P}Q \). Output \( Q = P(C, \omega, L) \), which is a nondecreasing, twice differentiable and concave function of capital \( C \geq 0 \) and labor \( L \geq 0 \). \( \hat{P} \) is the random price with finite mean \( \bar{P} \) and variance \( V \) and zero higher moments. The firm is a price taker in the output and in the factor markets. We talk of labor as one entity.

The firm confronts two types of labor contracts:

A. The straight (spot) wage contract: Labor is hired at market wage \( w \). In this case the profit function is

\[
\hat{\tau}_A = \hat{P}F(C, L) - wL - rC
\]  

We refer to this as contract A.

B. The profit sharing contract: Labor gets a function \( (1-s) \), \( se(0, 1) \), of the firm’s profit at the end of the production cycle and a guaranteed component \( dw \), \( de(0, 1) \), as wage payment per hour. We call \( (s, d) \) the profit sharing structure. The profit sharing structure is agreed upon before the production run and assumed given. The profit function in this case is

\[
\hat{\tau}_B = s[\hat{P}F(C, L) - dwL - rC].
\]  

We refer to this as contract B\((s, d)\). The expected value of \( \hat{R}_A \), \( \bar{R}_A \) and its variance \( V_A \) are, respectively:

\[
\bar{\tau}_A = \bar{P}F(C, L) - wL - rC
\]
\[ V_A = F^2v. \]  \hspace{1cm} (4)

Those of \( \bar{\pi}_s \) are, respectively,

\[ \bar{\pi}_s = s[\bar{P}P(C, L) - d\bar{w}L - rC] \]  \hspace{1cm} (5)

\[ V_s = s^2F^2v. \]  \hspace{1cm} (6)

Which of the two contracts A and B will dominate the other in terms of delivery of welfare to the two players? To address this issue, we let the utility function of the firm (employer) and worker be, respectively,

\[ U^i_{\pi_i}(\bar{\pi}_i, V_i) = \bar{\pi}_i - (a_{i/2})V_i, \quad i = A, B \]  \hspace{1cm} (7)

\[ U^i_{\pi_i}(\bar{V}_i, V_i) = \bar{V}_i - (a_{w/2})V_i, \quad i = A, B \]  \hspace{1cm} (8)

where \( i \) refers to the labor contract, \( Y_i \) is worker expected income under contract \( i \), \( V_i \) is the variance of \( \hat{Y}_i \), \( a_i > 0 \) and \( a_i > 0 \) are employer's and worker's, respectively, constant risk aversion. This utility function is commonly employed because of its simplicity (Holmstrom and Milgrom, 1987; Aoki, 1984). Since both \( \bar{\pi}_i \) and \( V_i \) are functions of \( C \) and \( L \), we write the composite utility function over \( C \) and \( L \) as \( U_{\pi_i}(C, L) \). Let

\[ (C', L') = \arg\max_{C, L} U_{\pi_i}(C, L). \]  \hspace{1cm} (9)

**Definition 1:** B(s′, d) strictly Pareto dominates A in the sense of pure risk sharing if (a) \( U^i_{s′} > U^i_{sA} \) and (b) \( U^w_{s′} > U^w_{sA} \), where \( U^i_{s} \) is the agent \( i \)'s utility, \( i = E, W \), in contract \( i \) = A, B(s′, d), all evaluated at \( (C', L') \).

Definition 1(a) says that there is a sharing structure \( s' \), given \( d \), so that the utility the employer can attain by switching to \( B \), while maintaining the optimal factor mix in \( A \), exceeds the best he gets in \( A \). In this case, we say the employer strictly prefers \( B(s', d) \) to \( A \). Likewise by Definition 1(b) the corresponding worker utility under this profit sharing contract dominates worker utility under a spot contract. The two utilities are evaluated at \( (C', L') \) to isolate the risk sharing feature of \( B(s', d) \). Note, however, that \( U^{w}_{sA} \leq U^{w}_{s''} \) which is the maximum that the employer can realize under \( B(s, d) \). The maximum utility \( U^{E}_{s''} \) of \( E \) at \( B(s', d) \) combines both the risk sharing and the factor demand feature of profit sharing. We claim that if by pure risk sharing (i.e., respecting \( (C^*, L^*) \)), \( B(s', d) \) dominates \( A \), the two parties will switch to \( B(s', d) \) and will find a factor mix which will preserve worker gain.

We will need the following:
Definition 2: B(s'd) strictly collectively dominates A in the sense of pure risk sharing if \([U_{BA'} + U_{WB}] = W_B > W_A = [U_{EA'} + U_{WA}]\) where \(U_{BA'}, U_{WB}\) are again, evaluated at \((C', L')\).

Clearly if \(W_B > W_A\), a pure switch from A to B(s', d) brings about a risk related welfare gain. If no such gain is in the cards \([W_B - W_A = 0]\), Pareto dominance in the sense of Definition 1 or even of a weaker one (i.e., \(U_{BA'} > U_{WA'}\) and \(U_{WB} > U_{WA}\) with at least one an equality) cannot be attained. Thus, Pareto dominance implies collective dominance but not vice-versa.

III. Pure Risk Sharing

From (9) and (7), the maximum of B in A is

\[ U_{BA'} = \pi_A' - R_A'a_s \]  \hspace{1cm} (10)

where \(\pi_A' = [PF(C', L') - wL' - rC']\) and \(R_A' = [F(C', L')^2v/2]\). Under A, workers realize guaranteed income \(Y_A'' = wL'\). Thus

\[ U_{WA'} = wL' \]  \hspace{1cm} (11)

Adding (10) and (11), we have:

\[ W_A = [PF(C', L') - rC'] - R_A'a_s. \]  \hspace{1cm} (12)

Substituting \((C', L')\) into \(U_{BB}(\cdot)\), we get

\[ U_{BB}(C', L') = \pi_B' - R_B'a_s s^2 \]  \hspace{1cm} (13)

where \(\pi_B' = s[PF(C', L') - dwL' - rC']\) and \(R_B' = R_B\) for \((C', L')\). (13) is not the highest utility the employer can get under B(s', d). This would be \(U_{BB}(C'', L'')\) where \((C'', L'') = \arg\max U_{BB}(C, L)\).

Substituting \((C', L')\) into \(U_{WB}(\cdot)\) gives

\[ U_{WB}(C', L') = dwL' + (1-s)[PF(C', L') - dwL' - rC'] - R_B'a_w(1-s)^2. \]  \hspace{1cm} (14)

Summing (13) and (14) gives

\[ W_B = [PF(C', L') - rC'] - R_B'[a_s s^2 + a_w(1-s)^2]. \]  \hspace{1cm} (15)

Subtracting (12) from (15), noting that \(R_A' = R_B'\),

\[ W_B - W_A = R_B'[a_w(1-s^2)][(a_s/a_w) - ((1-s)^2/(1-s^2))]. \]  \hspace{1cm} (16)

(16) is the pure risk sharing value-added of profit sharing at \((C', L')\). Now \([1 - s^2] > 0\), \(R_B' > 0\) and
(i) \[ \lim_{s \to 1} \frac{(1-s)^2}{(1-s^2)} = 0 \]  

(ii) \[ \lim_{s \to 0} \frac{(1-s)^2}{(1-s^2)} = 1. \]  

We have shown the following:

**Proposition 1:** B(s, d) strictly collectively dominates A in the sense of pure risk sharing iff \( \left( \frac{a_d}{a_w} \right) > \left[ \frac{(1-s)^2}{(1-s^2)} \right] \).

Our main concern is whether profit sharing as risk sharing is attractive when the employer is less risk averse than workers \( \left( \frac{a_d}{a_w} < 1 \right) \). From Proposition 1 and (17i), the following is obvious:

**Proposition 2:** For any \( \left( \frac{a_d}{a_w} < 1 \right) \), there exists an \( s' \in (0, 1) \) so that B(s', d) strictly collectively dominates A in the sense of pure risk sharing.

Thus, as long as the employer is risk averse \( (a_d > 0) \), however slight, profit sharing can be made collectively more attractive than a spot contract. The more risk averse are the workers relative to the employer, the closer to 1 is the sharing parameter s for risk sharing advantage. A risk neutral employer will, of course, reject the risk sharing \( (s = 0) \). Risk-neutral workers (though unlikely) combining with a risk averse employer will always guarantee its positive contribution. More risk averse employer than employee also always guarantees \( \bar{W}_d > \bar{W}_w \). Collective dominance does not guarantee Pareto dominance. Agents are more concerned with Pareto dominance than collective dominance since it guarantees their individual welfare. What is the risk profile that guarantees Pareto dominance? We have

**Lemma 1:** There exists a \( s' \in (0, 1) \) such that the employer strictly prefers B(s', d) to A at \( (C', L') \).

**Proof:** (if) (a) We show that \( U_{sa} > U_{sa} \). Consider (10) and (13). For \( (C', L') \), \( \bar{r}_A' > \bar{r}_A' - \bar{r}_C' \) since \( 0 < d < 1 \). At the same time \( R'_a > R'_a s^2 \) since \( 0 < s < 1 \) and \( R'_a = R'_a \) at \( (C', L') \). At \( (C', L') \), the risk burden for the employer is larger in A than in B(s, d) but the gross profit in B is larger. The employer’s share of the gross profit rises as s rises. Let \( s = 1 \). Then there exists an \( s = s' < 1 \) such that \( s' \left( \bar{r}_C'(C', L') - \bar{r}_C' \right) > \bar{r}_C' \). At \( s' < 1 \), \( R'_a > R'_a (s')^2 \). Thus, at \( s' < 1 \), \( U_{sa} > U_{sa} \), \( \bar{r}_C' \). Q.E.D.

**Proposition 3:** Suppose for some \( s' \in (0, 1) \), the employer strictly prefers B(s', d) to A. Then the worker will
strictly prefer \( B(s', d) \) to \( A \) if and only if \( \left( \frac{a_w}{a_u} \right) > \left( 1-s' \right)^2/(1-s'^2) \).

**Proof:** We now show that \( U_{\pi_A} > U_{\pi_A} \). This is true if and only if from (11) and (14), we have

\[
(1-d)wL' + (1-s')[(\tilde{PF}(\cdot) - dwL' - rC') - R^*_u a_u (1-s')^2] > wL'.
\]

Simplifying we have:

\[
[\tilde{PF}(\cdot) - dwL' - rC'] - s'[\tilde{PF}(\cdot) - dwL' - rC'] - R^*_u a_u (1-s')^2 > (1-d)wL'.
\]

(18)

Now let the employer strictly prefer \( B(s', d) \) to \( A \). We have

\[
s'[\tilde{PF}(\cdot) - dwL' - rC'] - R^*_u a_u (s')^2 > \pi_A^* - R^*_u a_u.
\]

Thus:

\[
s'[\tilde{PF}(\cdot) - dwL' - rC'] > \pi_A^* - R^*_u a_u + R^*_u a_u (s')^2.
\]

(19)

Substituting the right hand side of (19) for the left hand side in (18) does not change the direction of the inequality. This, however, gives

\[
[\tilde{PF}(\cdot) - dwL' - rC'] - \pi_A^* + R^*_u a_u - R^*_u a_u (s')^2
\]

\[
- R^*_u a_u (1-s')^2 > (1-d)wL'.
\]

But \( [\tilde{PF}(\cdot) - dwL' - rC'] - \pi_A^* = (1-d)wL' \). Thus, we have:

\[
R^*_u [a_u (1-s'^2) - a_u (1-s'^2)] > 0.
\]

Simplifying, we have

\[
R^*_u a_u (1-s'^2) \left[ (a_w/a_u) - ((1-s')^2/(1-s'^2)) \right].
\]

(20)

Thus, the Pareto dominance of \( B(s', d) \) over \( A \) is established if the condition holds. (only if) We show this by contradiction. Suppose \( (a_w/a_u) < (1-s'^2)/(1-s'^2) \). From Proposition 1, we know that \( B(s', d) \) is not collective welfare-improving in the sense of pure risk sharing. If workers do better in \( B(s', d) \), the employer will necessarily do worse and vice-versa. Thus, strict Pareto dominance of \( B(s', d) \) over \( A \) is impossible.

Q.E.D.

Again in view of Proposition 3 and (17i), we have

**Proposition 4:** For any \( 0 < (a_w/a_u) < \infty \), there exists an \( s' \in (0,1) \) so that \( B(s', d) \) strictly Pareto dominates \( A \) in the
The probability of a layoff in a T is thus

\[ p = \frac{\text{negative profit}}{\text{positive profit}} \]

or

\[ 0 < \frac{\text{negative profit}}{\text{positive profit}} \]

Let the probability of layoffs be \( \frac{p}{T} \) and layoffs occur in a layoff cycle when the firm realizes there is a trade-off between income variance and layoff risk. Thus, risk-averse workers may be happier in a layoff in B. Thus a layoff at T has a benefit (1989) (Padoue and Sonnino, 1987) that comes with a positive share than with the lack of risk-averse workers are less concerned with income.

It may turn out that workers are less concerned with income.

IV. Risk of Layoff

Also guaranteed Pareto dominance.

This is also guaranteed Pareto dominance.

Thus, we have [\( T-S-T \) / \( T-S-T \) ] < [\( s \) / \( S-S-T \) ]

This concludes the proof of Proposition 1 and 2.

The following follows from Equation 1 and 2:

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for given \((C, L)\). \(G(\cdot)\) is monotonically increasing in its argument. Under a contract \(B\), no layoff will occur as long as
\[
P \geq \frac{(dL + rC)}{F(C, L)}
\]  
(22)
and the probability of layoff is
\[
u_B = \frac{G(dL + rC)}{F(C, L)}
\]  
(22)
for given \((C, L)\). Thus we have
\[
\text{Lemma 2: For any given } (C, L), \ u_B < u_A.
\]
We now modify the utility function of workers to linearly include layoff risk. Under \(A\) and adopting \((C', L')\) in (9), (11) now becomes
\[
U_{WA}' = wL' - u_A' \alpha
\]  
(23)
where \(u_A'\) is defined over \((C', L')\) and \(\alpha > 0\) is the weight given by workers to layoff risk. Under \(B\) for the same \((C', L')\), (14) now becomes
\[
U_{WB}' = (1-d)wL' + s'[PP(\cdot) - wL' - rC']
\]
\[- \alpha u_B' (s')^2 - u_B' \alpha
\]  
(24)
where \(u_B'\) and \(PP(\cdot)\) are defined over \((C', L')\). For \(U_{WB}' - U_{WA}' > 0\), we now have in lieu of (20):
\[
R_B' a_m (1-s'^2)[\frac{(a_B/a_m)}{(1-s'^2)/(1-s'^2))} + \alpha (u_A' - u_B') > 0.
\]  
(25)
(25) approaches (20) as \(d \to 1\) \((u_A' - u_B')\). The following is now obvious:

**Proposition 5:** If workers' utility falls with layoff risk \((\alpha > 0)\), \(B(s', d)\) strictly Pareto dominates \(A\) if either (a) \((a_B/a_m) \geq (1-s'^2)/(1-s'^2)\), or (b) if \(\alpha\) is high enough.

Note that the condition (a) in Proposition 5 no longer is necessary as it was in Proposition 3. In analogy to Proposition 4, we have Proposition 5(b). But Proposition 5(b), in contrast, allows \(E\) to be risk-neutral, i.e., \(a_B = 0\), as long as workers are concerned enough with layoff risk.

When layoff risk is of importance to workers, the importance of the relative sizes of risk aversion measures diminish.
Anecdotal accounts of Japanese worker attitude, for example, reveal a very pronounced preference for stable employment. This gives rise to lower entry point wage offers from large established corporations than entry point wage offers from small businesses (Aoki, 1988). Large corporations' workers' take-home pay typically consists of a basic wage and periodic bonus (30-40 percent (Nakamura and Nakamura, 1991)). Although for employee morale, the bonus levels did not quite vary with variations in profit in normal times (Mizuno, 1985; Ohashi, 1989), the bonuses were the first to be sacrificed in abnormal times such as during the deep recession in early 1990's. This allowed large corporations to avoid massive dehiring which confirms worker expectation on the benefit of the bonus scheme.

Summary

The risk sharing angle of profit sharing seems to have been shunted aside by the lively debate centered on the macroeconomic outcomes of M. Weitzman's proposal. It seems fruitful to divorce consideration of this pay scheme from the Weitzman crusade. The question is, as Simon (1991) remarked about cooperation in organizations, why there is anything besides a wage contract. Profit sharing persists and may even be gaining ground in various guises such as "bonus scheme."

We have shown that as long as the employer is risk averse, however, slightly, there is always a profit sharing contract that will collectively and Pareto dominate the spot wage contract. The employer's risk aversion can thus be smaller than the worker's risk aversion as it should be. The smaller is the employers risk aversion relative to that of the workers, the smaller is the extent of profit sharing. Since employers are generally less risk-averse than workers, profit sharing of workers should generally be less than 0.5 and rather small.

If layoff risk is of paramount importance to workers (i.e., employment stability in the literature), the attraction of profit sharing is enhanced even with a risk neutral employer and workers will be more open to variants of profit-related pay. This aspect seems to have been very pronounced among Japanese workers. Thus profit-related pay has organizational value-added independent of claimed macroeconomic outcomes.
References


