Discussion Paper No. 9305

August 1993

On Keynes' Aggregate Supply Function

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Abstract

The usual interpretation of Keynes' aggregate supply function is that it is based on the ordinary supply function where the output supplied is a function of price such that its marginal cost is equated to price. In this paper it is argued that Keynes' concept is based on Marshall's supply function where the supply price is a function of quantity and the supply price is average cost including normal profit.
On Keynes' Aggregate Supply Function

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The question is this: what would be a correct interpretation or reading of Keynes' aggregate supply function (AS function for short)? This has been the subject of an extensive literature (see e.g. the bibliographies in de Jong (1954) and Wells (1960)). The following excerpts from Keynes (1936) give the background:

... call the aggregate income (i.e. factor cost plus profit) resulting from a given amount of employment the proceeds of that employment. On the other hand, the aggregate supply price of the output of a given amount of employment is the expectation of proceeds which will just make it worth the while of the entrepreneurs to give that employment. (p. 24)

Let \( Z \) be the aggregate supply price of the output from employing \( N \) men, the relationship between \( Z \) and \( N \) being written \( Z = \Theta(N) \), which can be called the Aggregate Supply Function. Similarly, let \( D \) be the proceeds which entrepreneurs expect to receive from the employment of \( N \) men, the relationship between \( D \) and \( N \) being written \( D = f(N) \), which can be called the Aggregate Demand Function.

Now if for a given value of \( N \) the expected proceeds are greater than the aggregate supply price, i.e. if \( D \) is greater than \( Z \), there will be an incentive to entrepreneurs to increase employment beyond \( N \) ... up to the value of \( N \) for which \( Z \) has become equal to \( D \). Thus the volume of employment is given by the point of intersection between the aggregate demand function and the aggregate supply function; for it is at this point that the entrepreneurs' expectation of profits will be maximised. (p. 25)
... we can equate the marginal proceeds (or income) to the marginal factor cost; and thus arrive at the same sort of propositions relating marginal proceeds thus defined to marginal factor costs as have been stated by those economists who ... have equated supply price to marginal factor cost.¹

¹For example, let us take \( Z_\eta = \Phi(N) \), or alternatively \( Z = W\Phi(N) \) as the aggregate supply function (where \( W \) is the wage-unit and \( WZ_\eta = Z \)). Then, since the proceeds of the marginal product is equal to the marginal factor-cost at every point on the aggregate supply curve, we have

\[
\Delta N = \cdots = \Delta Z_\eta = \Delta \Phi(N),
\]

that is to say \( \Phi'(N) = 1 \); provided that factor cost bears a constant ratio to wage cost ... (p.55)

There are three minority views as to the correct reading of the AS function. (i) To Hawtrey (1956), \( Z = D \), which makes him a "classical" in Keynes' use of this term. (ii) To Patinkin (1976), \( Z \) is total variable cost, which interpretation has the difficulty that the expectation of profit is not maximized at \( Z = D \). (iii) To Dillard (1948), \( Z \) is total cost. We postpone comment on this until later.

The majority view (see e.g. Casarosa (1981), Chick (1983), Davidson and Smolensky (1964), De Jong (1954), Harcourt (1977), Marty (1961), Miller (1972), Nevile (1992), Roberts (1978), Robertson (1955), Tarshis (1979), Vandenborre (1958), Weintraub (1957), and Wells (1960)) is based on the standard supply function of purely competitive firms where the unit price \( p \) is equated to marginal cost, which we will call the \( p = mc \) reading. Let
real aggregate output \( Y = g(N) \) so \( N = g^{-1}(Y) = h(Y) \) and therefore \( h'(Y) \) is marginal cost in wage units. Then the majority view is that \( h'(Y)g(N) \) is the AS price corresponding to \( N \).

The main problem is that if \( N \) is the only variable factor, which in effect is the case with the footnote on p. 55 cited above where \( \Phi'(N) = 1 \), one has \( \Phi'(N) > 1 \) with the \( p = mc \) reading, for

\[
\Phi'(N) = h'(Y)g'(N) + g(N)h''(Y)g'(N) > 1
\]

since \( h'(Y)g'(N) = 1 \) (because \( h'(Y) \) and \( g'(N) \) are reciprocals) and \( h''(Y) > 0 \) (increasing marginal cost). Thus there is a conflict between the majority view and the footnote.

Nevile (1992, p. 256) says, however, that "the footnote is simply an error by Keynes." Nevile claims that the footnote can be correct only if \( p = \text{const} \), which is clearly not the case. It is easy to see that if \( p = \text{const} \), then

\[
\Phi'(N) = \frac{d(pY)}{dN} = p \frac{dY}{dN} + Y \frac{dp}{dN}
\]

reduces to

\[
\Phi'(N) = p \frac{dY}{dN} = h'(Y)g'(N) = 1
\]

with \( p = mc \). But this does not prove Nevile's claim.

There are other problems with the \( p = mc \) reading, even though Keynes (p. 44) did say that the AS function is basically the "ordinary supply function" in the aggregate: (1) Why should Keynes (p. 283) have to say
"Provided we can assume that the price is equal to the marginal prime cost, we then have ..." if, indeed, \( p = mc \)? He would have said "Since we can assume that ..." instead. Notice also the distancing tenor of his remark about "those economists who ... have equated supply price to marginal factor cost:"

(ii) The independent variable in the usual supply function where \( p = mc \) is \( p \), while in the AS function the independent variable is \( N \) or, equivalently, \( Y \).

The puzzle is solved by a few lines from Marshall:

--"The price required to call forth the exertion necessary for producing any given amount of a commodity, may be called the supply price for that amount."

--"The normal supply price "is that the expectation of which is sufficient and only just sufficient to make it worth while for people to set themselves to produce that ... amount."

--"Profits are a constituent element of normal supply price."

--"When ... the amount produced ... is such that the demand price is greater than the supply price, then sellers receive more than is sufficient to make it worth their while to bring goods to the market to that amount; and there is at work an active force tending to increase the amount brought forward for sale."

These four statements are from pages 142, 373, 618 and 345, respectively, of Marshall (1920). Notice the similarity with the excerpts from Keynes quoted above. To Keynes, the "ordinary" supply function is clearly that of Marshall, not the currently standard one based on \( p = mc \).
and for good reason. Since production takes time, the production decision must depend on the entrepreneur's expectation of the price when the output does get to market (not on the "given" current price), and the supply function tells the price which, if expected, barely induces him to produce any given amount. The price expectation that will turn out to be correct is, of course, the demand price, i.e. the price at which the given amount can be sold. Entrepreneurs will therefore employ that number of men whose output has a supply price equal to their expectation of the demand price, and if expectations turn out to be correct, one has an equilibrium.

It also seems quite clear that in Marshall, the normal supply price is the average cost plus normal profits, or simply average cost; the latter is interpreted to include normal profits.

Going back to the footnote example where the ratio of factor cost to wage cost is assumed to be a constant α (say), factor cost can be calculated from wage cost merely by multiplying the latter by α. Suppose then that N is the only variable factor. Letting \( \pi^* \) denote the minimal profit required by entrepreneurs, the AS price is

\[ \Phi(N) = \text{fixed costs} + \pi^* + h(Y) \]

and therefore

\[ \Phi'(N) = h'(Y) \frac{dY}{dN} = 1 \]

as stated in the footnote. Taking all factors into account, \( \Phi'(N) = \alpha \) but, as Keynes might have said, this is obvious. (After Robertson sent Keynes his comments on a galley proof of the General Theory, Keynes wrote back regarding
one item: "Perhaps I should have said ... But isn't this obvious?" See Moggridge (1973, p. 316).

Entrepreneurship and management are factors that can vary, and it is reasonable to expect required profits to be higher with higher $N$. Thus indeed, the expectation of profit is maximized at the point where $Z = D$.

Finally, there is one false issue to dispose of. Since Keynes accepted the classical postulate that labor gets paid its marginal product, it might seem that he was committed to the $p = \frac{m}{m}$ view. For if $N$ is the only variable factor, then the condition $p = \frac{m}{m}$ is equivalent to this postulate (because $p^1$ is the real wage in wage units and $p^1 = \frac{dY}{dN}$ can be written $p = h'(Y)$). However, there are other variable factors of course, and the Marshallian supply price is average cost (including normal profits). One can therefore accept the postulate which is a necessary condition for profit maximization no matter how $p$ is determined, and reject $p = \frac{m}{m}$.

We draw the following conclusions:

(1) The Marshallian supply function makes more sense than the $p = \frac{m}{m}$ supply function. In Marshall and in Keynes, quantity adjustments are made towards equilibrium when price expectations happen to be wrong. In the usual treatment, prices are "given" to firms so there is no guesswork, entrepreneurs mechanically maximize profit by choosing that amount of output whose marginal cost equals the given price, production is in effect instantaneous, and a fictitious auctioneer is needed to cry out tentative prices until equilibrium is reached.

(2) Most, if not all, of Keynes' commentators have misread him on the
AS function. This is somewhat surprising since Keynes is often reported to be Marshallian and not Walrasian. In all the literature on the AS function, only Dillard (1948) took Keynes' definition of the AS price to mean total cost. However, Dillard did not refer to Marshall and did not state that total cost included normal profits. In any event, Dillard's reading was either ignored in the subsequent literature or judged (see de Jong (1954)) to be incorrect.

(3) Since the Marshallian supply price is lower than \( p = mc \), price is lower and output higher than in the standard model.

(4) Lastly, normal profits generally obtain not only in long period equilibrium but also in the short period. That is, when entrepreneurs' expectations about demand are correct, they make no more than normal profits. (The setting is one of purely competitive firms.) This implication seems to be particularly interesting.
Notes

1. Paper presented as part of the festschrift lecture series in honor of José V. Abueva, University of the Philippines, 1993.

2. The author is indebted to Ruperto Alonzo, Arsenio Balissacan, Emmanuel de Dios, Raul Pabellón, Felipe Medalla and Rafael Rodriguez for helpful discussions.
References


