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Aggregate Supply: A Reformulation

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Abstract: This paper proposes an aggregate supply function based on monopolistically competitive firms without assuming a Phillips curve relation. With aggregate demand the resulting framework can accommodate Phillips curves, stagflation, and procyclical real wages. There is a range of potential full-employment output levels, and interestingly, from an unemployment equilibrium it may be possible to reach full employment with a higher real wage.

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1. Introduction

Dornbusch and Fischer (1990, p. 471) have correctly observed that "The theory of aggregate supply is one of the least settled areas in macroeconomics." They derive an aggregate supply curve but they assume a Phillips curve relationship in order to do so, which is methodologically unsatisfactory. Phillips curves ought to be explained and not assumed, especially since they have failed to hold at times, as in the 1970s. Assuming monopolistically competitive firms, this paper proposes an aggregate supply function that does not presuppose a Phillips curve relation. In conjunction with aggregate demand the resulting framework can explain the Phillips curve when it does hold, and also stagflation when this occurs. The novel result is a range of possible full-employment levels of output, which has interesting implications.

2. The firm's price-output decision

Suppose a monopolistically competitive firm's differentiated product has a demand function \( x = x(p, \alpha) \) where the price \( p \) is set by the firm and \( \alpha \) is a demand parameter with \( x_p = \frac{\partial x}{\partial p} < 0 \) and \( x_\alpha > 0 \). (Other prices and determinants of demand are reflected through \( \alpha \).) Let \( k(x, w) \) be the variable cost of producing \( x \), with \( w \) a cost parameter and \( k_w > 0, k_x > 0, k_{xw} > 0 \). Maximizing profit, the firm maximizes \( px(p, \alpha) - k(x(p, \alpha), w) \) which requires that

\[
(p - k_x)x_p + x = 0 \quad (1)
\]

\[
(p - k_x)x_{pp} + 2x_p - k_{xx}x_p^2 < 0. \quad (2)
\]
We assume sufficient competition in the economy—this makes the slope of the demand curve relatively small with quantity on the horizontal axis as usual—such that at the firm's profit-maximizing output level, marginal cost is nondecreasing: \( k_{xx} \geq 0 \). Writing \( D \) for the left-hand side of (2), total differentiation of (1) gives

\[
D \, dp + \left( (p - k_x) x_{p \alpha} + (1 - x_p k_{xx}) x_{\alpha} \right) d\alpha
- x_p k_{xw} \, dw = 0,
\]

Hence

\[
\frac{\partial p}{\partial \alpha} = \frac{\left( (p - k_x) x_{p \alpha} + (1 - x_p k_{xx}) x_{\alpha} \right)}{D} > 0
\]

if, since \( x_{p \alpha} \geq 0 \) might be expected, \( x_{\alpha}/(p - k_x) > |x_{p \alpha}| \), which is more likely the greater is the degree of competition as this makes \( p - k_x \) smaller. Thus, assuming enough competition when \( x_{p \alpha} \neq 0 \), (4) holds. Also,

\[
\frac{\partial p}{\partial w} = x_p k_{xw}/D > 0.
\]

Summarizing (4) and (5), \( p \) is higher if \( \alpha \) or \( w \) is higher. With \( \alpha \) and \( w \) the only parameters in the price decision, this gives

**Proposition 1.** The monopolistically competitive firm will set a higher price if and only if the demand parameter \( \alpha \) or the cost parameter \( w \) is higher.

A rightward shift of the demand curve implies a similar shift of the marginal revenue curve, so the latter must intersect the marginal cost curve at a higher value of \( x \). Thus with higher demand, the firm will produce more output.
The firm's supply function takes \( w \) as given. With \( w \) given, Proposition 1 says that price is higher only if demand is greater. Since more output will be produced with greater demand, this implies

**Proposition 2.** The firm's supply curve \( 3/ \) is upward-sloping: more output will be supplied at a higher price.

To see the effect of an increase in the cost parameter on the supply curve, we assume (following Dornbusch and Fischer) that the short-run production function is simply \( x = qL \) where \( L \) is labor, so that \( k(x, w) = wx/q \) where \( w \) is the money wage rate. Denoting by \( x^* \) superscripts evaluations at the firm's price-output point, a Taylor linear approximation gives

\[
x(p, \alpha) = x^* + (p - p^*)x^*_p + (\alpha - \alpha^*)x^*_\alpha.
\]

i.e. \( x^* = x(p^*, \alpha^*) \) with \( \alpha^* \) the existing parameter value. For notational simplification write

\[
x^* - p^*x^*_p - \alpha^*x^*_\alpha = A
\]

\[-x^*_p = s \quad x^*_\alpha = r.
\]

Since the unit of measurement of \( \alpha \) is arbitrary, we can choose units to set \( r = 1 \) so that

\[
x(p, \alpha) = A - sp + \alpha
\]

and (1) becomes
\[ p = \frac{A}{2s} + \frac{\alpha}{2s} + \frac{w}{2q}. \]  

(1a)

We can state

**Proposition 3.** A higher (lower) \( w \) will shift the supply curve upwards (downwards) but proportionately less than the increase (decrease) in \( w \).

**Proof.** Consider a \( dw \) increase in \( w \). This will shift the supply curve upwards a vertical distance of say \( dp \) which is the sum of two components: (i) the increase \( d_1p \) resulting from moving upwards/leftwards along the demand curve to its intersection with the new supply curve, which reduces output by the amount \( dx \) say; (ii) \( d_2p \) given by \( dx \) times the slope of the new (the same as that of the old) supply curve.

(i) Putting \( dw = 1 \), \( d_1p = \frac{\partial p}{\partial w} = \frac{1}{2s} \). Since the slope of the demand curve is \(-1/s\), \( \frac{1}{2q} = (-1/s)(-dx) \) or \( dx = s/2q \).

(ii) The slope of the supply curve is \( \frac{\partial p}{\partial q} = \frac{1}{2s} \), so \( d_2p = \frac{1}{4q} \).

Thus \( dp = \frac{3}{4q} \), and \( dp/p = \frac{3}{4qp} \) is to be compared with \( dw/w = 1/w \). Now \( p > \frac{k_x}{w} = \frac{w}{q} \) or \( qp > w \), so of course \((4/3)qp > w\), i.e. \( dp/p < \frac{1}{4q} \). If \( dw < 0 \) whence \( dp < 0 \). \( |dp/p| < |dw/w| \) also follows.

The importance of Proposition 3 will become clear in due course.

3. Aggregate functions and equilibrium

**Aggregate supply**
The general aggregation problem is not the topic of this paper and we avoid it by assuming that the firm in Section 2 is the representative firm. Proposition 2 then gives an aggregate supply curve like AS in Fig. 1. Write this relationship as

\[ Y = f(p|w, \ldots) \]  

(6)

where \( Y \) is real aggregate output and \( p \) is now the price level, the money wage \( w \) and other variables being given. In particular, \( w \) will be taken as a bargaining outcome which can change periodically. (If needed, \( w \) can be interpreted as a composite variable that includes the costs of material inputs assumed to be required in fixed proportions with labor; otherwise, \( w \) is simply the money wage.)

One can also have an upward-sloping AS if firms are purely competitive, but the latter assumption has the well-known difficulty that if everyone is a price-taker, there is no one to set prices, the Walrasian auctioneer being a fiction. Price-setting firms are needed if prices are to change.

**Aggregate demand**

An aggregate demand formulation based on the IS-LM model—see e.g. Dornbusch and Fischer (1990)—is quite standard. With the money stock \( M \) exogenous, one can write it as

\[ Y = g(p|M, \ldots) \]  

(9)

giving the aggregate demand schedule labelled \( AD \) in Fig. 1. (\( AD^C \) and \( AD^f \) should be ignored for the present.)
Equilibrium

AD tells the \((p, Y)\) combinations satisfying the IS and LM equilibrium conditions, IS merely saying that output equals demand. In order for the output to obtain, firms must be willing to produce it, which is the information provided by AS. Denoting by \((p^e, Y^e)\) the solution to the pair of equations (6) and (7), the equilibrium is at point \(e\) in Fig. 1.

The economy will always be on AS because price and output decisions are made by firms and they know AS. However, they can be wrong in their estimate of demand. Suppose the economy is at point \(a\). With output/income \(Y^a\), AD says that \(p\) needs to be higher (at \(d\)) in order for the goods and money markets to clear. The price at \(a\) being too low, there is excess demand for goods which signals firms to raise their prices and increase their outputs. In the opposite case where the economy is at \(c\), there is excess supply of goods. The arrows point towards the stable equilibrium \(e\).

Two properties of the model make for relatively straightforward comparative statics. First, as already noted above, the economy is always on AS. Second, there is a close connection between the micro and macro levels: \(\alpha\) will increase if and only if there is a rightward shift of AD, and AS is shifted upwards if and only if \(w\) is higher. Thus, depending on which is more convenient, one can speak of shifts in AD and AS, or changes in \(\alpha\) and \(w\). From (1a),

\[
dp = d\alpha/2s + dw/2q \tag{1b}\]
\[ \frac{dp}{p} = (\alpha/2sp)d\alpha/\alpha + (w/2qp)dw/w \]  

(1c)

so \( \frac{dp}{p} \) is a linear combination of \( d\alpha/\alpha \) from the demand side and \( dw/w \) from the cost side. Because AS is upward-sloping, one can then also find the change in \( Y \).

4. Implications

The first question is whether there is full employment at \( e \). Let \( L^S = L^S(w/p) \) be the labor supply function and write

\[ Y = h(p|w) \]  

(8)

for the output \( Y = Y(L^S) \) that \( L^S \) can produce. Representing (8) by LS in Fig. 1, labor supply in output terms can then be depicted along with AS and AD in the same diagram. Let \( L^d = L^d(p|w, \ldots) \) be the demand for labor. Expressing this similarly in terms of the output it can produce, \( Y = Y(L^d) = f(p|w, \ldots) \), i.e. the demand for labor in output terms is identically (6), because firms hire only that amount of labor needed to produce the output they are willing to supply. Denoting by \( (p^C, Y^C) \) the solution to (6) and (8), full employment is defined at point \( c \) in Fig. 1 given \( w \), with \( Y^C \) the corresponding output. Since (6), (7) and (8) are mutually independent, \( Y^e \neq Y^C \) in general. Postponing discussion of \( Y^e \geq Y^C \) possibilities, consider first the \( Y^e < Y^C \) situation of Fig. 1 showing \( e \) as an unemployment equilibrium.

Phillips curves

Assume the economy is normally at \( e \) but there are random shocks that shift AD rightwards, after which the economy returns to \( e \). The
larger the shift, the greater is the increase of $p$ over its previous value and the higher is $Y$, hence the lower is the unemployment rate. With a corresponding remark holding for leftward shifts of $AD$, the result would amount to the Phillips curve relationship as usually stated: $dp/p$ and the unemployment rate are negatively correlated. However, as Phillips (1958) observed, the time pattern of the unemployment rate is not random but often conforms to the business cycle.

Start from the trough of a business cycle. An increase in $\alpha$ that initiates the upswing will raise $p$. With more unemployment than usual, there may be no pressure initially from employed workers to raise $w$, but in time they are likely to press for a higher $w$ in order to protect their real wage as $\alpha$ and $p$ continue to rise. A higher $w$, however, raises $p$ further. Even if each period's $da/\alpha$ is the same during the whole course of recovery and expansion, the rising $w$ in the expansion and especially in the end phase will therefore raise $p$ increasingly more. In macro terms, even with the same relative shift of $AD$ each period but an increasing $w$ when there is less unemployment, which induces increasingly larger shifts in $AS$, increases in $p$ will be accentuated while increases in $Y$ will be attenuated nearing the peak. This suffices to account for the characteristic nonlinearity of the Phillips curve, since movements from peak to trough might be expected to follow roughly the same process in reverse. The Phillips curve is thus explainable by shifts in $AD$ and their concomitant effects, including shifts in $AS$ induced by changes in the money wage.

As was early noted by Lipsey (1960), money wage and price level
changes are highly correlated. The reason is clear from (1c) where only
the term involving $\alpha$ prevents the correlation from being a perfect one.
The original (money wage) Phillips curve can therefore be explained
similarly.

The real wage

This raises the question of the behavior of the real wage in the
business cycle. Bodkin's (1969) empirical study indicated that the real
wage rose more often than it fell in the upswings, and the evidence now
seems to favor the idea that it is "slightly procyclical" (Fischer, 1980,
p. 310) or at least that the real wage and the employment level are
independent (Geary and Kennan, 1982). Such conclusions are difficult to
reconcile with the usual assumption of price-taking firms equating the
real wage to the marginal product of labor with the latter decreasing with
more employment in the short term. In the framework of this paper, the
issue depends simply on the relative magnitudes of the changes in $\alpha$ and
$w$. In (1c) write $\alpha/2\sigma p = B$ and $w/2\sigma p = C$:

$$dp/p = B \frac{\alpha}{\alpha} + C \frac{dw}{w}. \quad (1d)$$

If $\alpha = 0$ and $dw > 0$, $dp/p = C \frac{dw}{w}$ so (recalling that $dp/p < \frac{dw}{w}$
from Proposition 3) $C < 1$. It then follows directly from (1d) that
$\frac{dw}{w} - dp/p > 0$ (the real wage will rise) if $\frac{dw}{w} > \frac{(B(1-C))}{\alpha}$,
which inequality may well hold in the end phase of an expansion as the
labor market gets increasingly tighter.
Stagflation

Stagflation is explainable along conventional lines (as in Dornbusch and Fischer) by upward shifts of AS.

Unemployment and full employment

The important implication of Proposition 3 at the macro level is that $dw$ has a smaller vertical shift effect on AS than on LS—the latter is shifted exactly proportionately—which means that in Fig. 1 their intersection point $c$ will move northeast with an increase in $w$ and southwest with a decrease. By lowering $w$ sufficiently, $c$ can therefore be made to move southwest until it touches AD at which point, say b, there is full employment. The practical difficulty is of course the resistance of employed workers to a cut in the money wage because this implies a reduction in the real wage. (From Proposition 3 the real wage is lower at the point in the new AS directly south of e, and since the $p$ level at $b$ is even higher, the real wage at $b$ is lower yet.) Alternatively, by means of active policy that changes the parameters underlying AD, AD can be made to shift rightwards until it touches $c$, giving full employment also but with a higher price level (hence a lower real wage) and more output. Since $b$ and $c$ are both full-employment points and employment is higher at $c$, the real wage there is obviously higher. Varying the mix of lower $w$ and higher AD gives a whole range of possible full-employment points with different real wage and output levels. (The natural rate of unemployment—Solow (1986) and others have questioned the usefulness of such a concept—is accordingly not well defined.) We can therefore state
Proposition 4. A range of full-employment levels of output can be generated by different money wage and aggregate demand combinations.

This puts the familiar distinction between classical unemployment, caused by an excessively high real wage, and Keynesian unemployment, due to deficient aggregate demand, in a different light. Neither cause is to be found without the other, for at any unemployment equilibrium like e the real wage is too high and aggregate demand is also too low. The important difference lies in the implied prescriptions for full employment. The Keynesian higher-AD route to c gives more output and employment at a higher real wage compared to the classical lower-w route to b. Intermediate full-employment points being possible, they only represent polar cases. The field of choice is the "triangle" defined by e, c and b, and since the real wage is highest at e, tradeoffs do not lie along a curve but over an area.\(^5\)

Using Fig. 1 again suppose that the economy is at c which is now an equilibrium because AD has been shifted to AD\(^c\), and another change in some parameter sends AD\(^c\) to AD\(^1\). The point e', where AS and AD\(^1\) intersect, is not feasible because there is not enough labor available to produce the output there. With excess demands for goods and labor at c however, firms can raise w (and therefore also the endogenous p) until a new full-employment equilibrium is reached on AD\(^1\) determined by the new LS' and AS' (not shown). Unless some ceiling on output has been reached—this requires either LS' or AS' to be perfectly inelastic at c—the new equilibrium will be northeast of c, say at c'. Thus
Proposition 5. Under conditions of full employment and given a higher AD, there exists a higher money wage that will increase output, employment and the real wage except when labor supply or aggregate supply is perfectly inelastic.

This implies that since the higher AD could be due to a higher M, money is not neutral in the short run even under full employment. Noting further that the real wage is lower at c than at the original e but higher at c' than at c, if the shift from AD to AD' is large enough the real wage at c' could be higher than at the original e. We can therefore state the following unusual proposition.

Proposition 6. Suppose an unemployment equilibrium. Unless some ceiling on output should prevent it, a two-stage increase in AD can achieve full employment with a real wage higher than at the unemployment equilibrium.

5. Conclusion

An upward-sloping aggregate supply curve (AS) can be defined in an economy of monopolistically competitive firms without assuming a Phillips curve relation. AS is also the demand for labor measuring the latter in terms of the output it can produce. Coupled with an aggregate demand schedule (AD), stagflation is explainable by upward shifts in AS, and the Phillips curve by shifts in AD. In line with observation the real wage may rise in the expansion phase of the business cycle. An increase in the money wage rate shifts AS relatively less than it does the labor supply curve, which implies a range of possible full-employment levels of
output. Given an unemployment equilibrium, full employment can be reached by lowering \( w \), raising \( AD \), or a mix of both, and a two-stage increase in \( AD \) can achieve full employment at a higher real wage unless this is prevented by some ceiling on output.

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Notes

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2. The assumption of price-setting firms as a basis for macroeconomic formulations has been used most recently by Lindbeck and Snower (1988) and Startz (1989); earlier references include Hart (1982), Malinvaud (1985) and others cited by Benassy (1987).

3. Some readers might balk at calling this a supply relationship because both price and quantity are decided by a monopolistically competitive firm and a narrow convention apparently takes it that only price-taking firms have supply curves. But this is only a matter of terminology. A supply curve simply tells "the relationship between the supply of a commodity and its price" (Pearce (1986), p. 408) so a monopolistically competitive firm can also be said to have a supply curve.

4. They might accept a reduction if by doing so they prevent some of their number from being laid off, but they are not likely to be greatly concerned with more hiring; see Lindbeck and Snower (1986) and Blanchard and Summers (1986).

5. Although it might seem the best choice this would mean a
preference for employing a larger work force at a lower real wage, those employed at e having a higher one. The choice of b where p is lower would favor retired workers with fixed pensions.
References


