Sources and Variability of Inflation
in an Open Economy

by

Carlos C. Bautista

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Sources and Variability of Inflation in an Open Economy

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Abstract

This paper presents an empirical analysis of inflation in an open economy. Using Philippine data, the paper first estimates the variance of inflation using a standard ARCH model and shows that periods of high inflation are succeeded by periods of high volatility. Next, the sources of inflation are analyzed using a macroeconomic vector autoregression model. The variance decomposition attempts to determine whether the fiscal view or the BOP view of inflation holds for the Philippines. It is shown that the BOP view holds but this result is not sufficient to say that the former is unimportant in the inflation process.
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I. Introduction

One of the central variables in macroeconomic policy debates is the inflation rate. The importance of this variable stems from the fact that inflation measures the relative price of present versus future commodities. Uncertainty in future prices affects present consumption behavior and therefore influences the manner by which resources are temporally allocated. An issue frequently addressed in theoretical discussions is the welfare implication of inflation. Traditional steady state models show that inflation has little or no effect at all in most real variables. Yet, these models demonstrate that even with perfect anticipation inflation can lead to welfare losses for individuals.¹

In reality, inflation moves irregularly and may at certain periods be unanticipated. Unanticipated jumps in the price level are, for individuals, periods of uncertainty where the possibilities of such losses are perhaps more immense than those predicted by models assuming perfect

¹Bailey (1956) first proposed measures of the welfare effects of steady inflation using the consumer surplus approach. This spawned the inflation tax literature.
foresight. This weakness of steady state models analyzing inflation was recognized by Marty (1976).

Changes in the money stock, the exchange rate or intermediate input prices all influence the general price level. However, a once-and-for-all movement in these variables by themselves cannot explain persistent inflation.¹

It is quite easy to recognize that stabilization policies which are themselves anti-inflation devices, do influence the inflation process. It is however analytically difficult to explain which of these stabilization policies contribute more to inflation variation. For open economies like the Philippines, the analytical distinction gets blurred as exchange rate policy is closely linked with fiscal and monetary management.

There are several notions as to the causes of inflation. The fiscal approach to inflation basically views monetary growth as the source of inflation. Further decomposing monetary growth, the ultimate source of inflation would be the exogenous fiscal deficit. This view of inflation does not seem to track the inflation pattern of

¹Protracted inflation though does occur with a continued devaluation or extended money growth both of which are policy directed.
highly open economies and countries which try to solve BOP difficulties through heterodox adjustment programs.

In this literature, researchers analyzing inflation in these countries instead believe that while monetary growth also contributes to price movements, the inflation patterns experienced can be largely attributed to either the nominal exchange rate movement or the incomes policies that accompany a heterodox adjustment program. The BOP view maintains that the exchange rate is the main cause of inflation. Inflationary episodes due to exchange rate adjustments occur when a nominal devaluation is accompanied by a passive monetary policy stance. For some Latin American countries like Brazil and Argentina, the unsustainability of wage-price controls after a certain period is believed to have led to inflation instability (Kiguel and Liviatan, 1990).

This paper addresses the empirical issues surrounding the theoretical discussions on inflation within the Philippine setting. While the Philippines and some Latin American countries suffer from a severe foreign exchange constraint, they differ in the manner by which problems of inflation instability emanating from this constraint are addressed. The Philippines did not rely on incomes policies as an adjustment mechanism and anti-inflation device. Overall macroeconomic policy however is used to impose
inflation discipline through monetary management and the maintenance of a particular exchange rate given a sufficient flow of foreign resources. Whether monetary growth or exchange rate movements contribute more to the inflation process is an empirical question. This paper is an attempt to determine whether the fiscal view or the BOP view of inflation holds for the Philippines.

The second section gives a brief history of Philippine inflation. The third section presents the empirical portion of the paper. The variance of Philippine inflation is estimated using a standard autoregressive conditional heteroscedasticity (ARCH) regression technique due to Engle (1982). Next, the dynamic relationships between inflation and other macroeconomic variables are explored using an empirical methodology adopted in Montiel (1988) where a test of whether the fiscal view or the BOP view holds in Argentina, Brazil and Israel. In the procedure used, the forecast error variance of inflation is decomposed using a macroeconomic vector autoregression (VAR) model to find out the relative degrees to which exogenous components of other variables influence inflation. The fourth section concludes.

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*It should be noted that insufficient foreign resources contribute to high inflation but does not explain inflation patterns.*
II. Philippine Inflation Experience

The Philippines has a long history of import and exchange controls. These controls were thought to complement an import substituting industrialization strategy in the 1950s until the end of the 1960s. Throughout this period, control measures restricted selected imports (machinery and intermediate inputs) to available foreign exchange receipts. Despite this, the current account was consistently in deficit as imports outstripped exports. The failure of controls exerted pressure on the balance of payments. The conscious effort to defend the exchange rate in effect, forced a devaluation in 1962 but did not cause serious economic dislocation. It in fact rectified a BOP imbalance as exports of agricultural goods were stimulated. The Philippines did not suffer a slowdown or double digit inflation because the industrial structure, as yet, was not so dependent on imported inputs. It was also a quiet era of relatively low oil prices.

The international economic environment dramatically changed with the onset of a world recession in the 70s and as oil prices increased. A devaluation in 1970 relieved the pressure temporarily. As a way of coping with a volatile external environment in the 1970s, the Philippines relied heavily on foreign borrowings to finance growth without undertaking corresponding structural adjustments.
particularly in the price of its currency. Inflation discipline at that time was imposed using a combination of the enforced exchange rate regime and accommodating short-run stabilization policies.

A "growth through foreign debt" strategy is unsustainable without sufficient structural change. As a result, the Philippines experienced its fourth balance-of-payments crisis in 1983. The episode of high inflation was preceded by a forced float of the exchange rate and severe restrictions on credit creation. By this time, more difficulties were encountered as the Philippines experienced negative growth rates for the first time in its history. During the last half of this decade, the Philippines experienced wild swings in the price level and undertook stabilization programs to control the rate of inflation. Up to the present, authorities are having difficulties in containing inflation.

III. Empirical Analysis

A. The Variance of Inflation

The volatility of inflation is measured using an ARCH model. This econometric procedure relaxes the assumption of constant unconditional variance of ordinary least squares (OLS) regression and makes a distinction between the
conditional and unconditional variance (See Appendix A for a description of the method).

A reduced form equation for inflation is used in this paper and the result presented in Table 1 was selected from a number of specifications after experimenting with lag lengths on the dependent variable.

The percentage rate of change of the GNP deflator was used as the inflation variable and is computed as the first difference of the deflator's logarithm. The exogenous variables are the lagged values of the percentage rates of change of narrow money (m), exchange rate (x) and real GNP (y). The wage rate was not included because the Philippines has a long history of legislated wages which continue up to the present time. A dummy variable was however included for years where GNP growth was negative. The final choice was made based on the highest adjusted $R^2$. The OLS result of the final model is shown in the first column of Table 1. The estimation is for the period 1950 to 1980.

Values in parenthesis are t-ratios. All slope coefficients are significant at the 10 percent level except for the second lag of the inflation rate. Following conventional procedures described in Appendix A, this final

*This reduced form equation may be viewed as reduced form derived from some structural model.*
equation was tested for the presence of ARCH effects. An application of the Lagrange multiplier test for different orders of the ARCH process was conducted. The test reveals the presence of ARCH effects only of the first order as shown by the Chi-square statistic. This is shown below:

\[ e_t^2 = 0.0013 + 0.3707e_{t-1}^2 \]
\[ (2.3907) \quad (2.4900) \]

\[ R^2 = 0.1371 \quad N = 41 \quad \text{Chi-squared}(1) = 5.624 \]

The parameter estimates in this equation are significant at the 5% level. Second and higher order tests which yielded no evidence of ARCH effects are not shown.

The second column of Table 1 shows the estimate of the same specification which recognizes the presence of first order ARCH effects. The results show significant improvements over the OLS estimates. Parameter estimates (\( \alpha_0 \) and \( \alpha_1 \)) in the variance equation are also highly significant.
Table 1
OLS and ARCH Estimates of Inflation

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<tr>
<td></td>
<td>(-0.6837)</td>
<td>(-0.5838)</td>
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<td>$p_t-1$</td>
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A derivative result of the ARCH procedure is an estimate of the inflation rate's variance for the sample period. This is shown graphically for the period 1950 to 1990. From the graph, the variance of inflation seems to follow the historical patterns of Philippine inflation.
Mild increases of the variance in 1952, 1954 and 1965 is immediately discernible from the plot.

Except for these spikes, one can see that 1950 to 1969 is a period of low volatility of inflation. The next two decades were however highly volatile periods. The variance of inflation peaked during the years after the oil crisis (1973) and after the fourth balance of payments crisis (1983). This result presents no evidence of high variability during high inflation episodes as elucidated in Friedman (1977). Instead, the uncertainty appears after two to three years.

B. Sources of Variation

This section explores the dynamic relationships between inflation and other important variables in the system that
are believed to influence the Philippine inflationary pattern. A macroeconomic VAR model is constructed and used in decomposing the forecast error variance of inflation. Error variance decomposition is a method by which one can show the relative importance of variables in influencing the movements of other variables in the system. A particular variable should contribute more to the forecast error variance of the variable of interest the more it exhibits variability in the sample period. VAR estimation is described in Appendix B (See also Sims, 1980).

As mentioned in the first section, the objective of this modeling exercise is to test the competing views as to the causes of inflation. In this model therefore, the exogenous components of two basic variables, monetary growth and exchange depreciation, are assumed to affect the inflation rate. To avoid possible mis-specification, income growth is included in the model. As in the ARCH model, the wage rate was not included for the same reasons stated above. The variables of the system then are the growth rates of narrow money (m), exchange rate (x) and real GNP (y) and the GNP deflator (p). An exogenous time trend was included in the system.

A VAR system may be described by the following set of equations:
2) \[ q_t = A(L)q_{t-1} + u_t \quad ; \quad A(L) = A_0 + A_1L + A_2L^2 + \ldots \]
\[ E(u_t) = 0 \]
\[ E(u_t u_s') = 0 \quad ; \quad t \neq s \]
\[ E(u_t u_s') = 0 \quad ; \quad t < s \]
\[ E(u_t u_t') = \Sigma \]

where \( q_t \) is an nx1 vector of variables, \( u_t \) is the nx1 error vector, \( L \) is a lag operator and has the property that \( L^k x_t = x_{t-k} \). For this paper, a first order VAR model was estimated. The choice of this lag length was based on Akaike's information criterion (AIC).\(^5\)

The VAR system was used to make a seven year ahead forecast for this variables. The forecast error variance of the inflation rate (\( p \)) is then decomposed into innovations attributed to itself and to other variables \( y, x \) and \( m \). The results are shown in Table 2 below. The system was estimated for different orderings to observe the sensitivity of the results to this statistical procedure.

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\(^5\)The AIC chooses the lag length which minimizes the quantity \( N \log(\text{det} \Sigma_k) + 2n^2 k \). \( N \) is the sample size, \( n \) is the dimension of the vector \( q \), \( k \) is the order of the VAR model and \( \Sigma \) is the covariance matrix of the residuals. The second term may be thought of as a penalty parameter as lag length increases.
Table 2
Forecast Error
Variance Decomposition of
the Inflation Rate

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The first thing to notice is that the effect of inflation's own innovation for the seven year period does not go below 50 percent but a drastic decline is seen in the second period from 81 percent to 55 percent. For the first year, the effect of supply shocks are larger than monetary or exchange rate innovations but declines as the forecast horizon increases to five periods. On the other hand, the innovations due to x and m are steadily increasing.
The results are not so sensitive to the ordering of the variables with respect to the effects of innovations in $x$ and $m$. Except for the ordering $y-m-x-p$, the contribution of $x$ innovations in the variance of $p$ are higher than those of $m$ for the whole forecast period.

IV. Concluding Remarks

The estimates of Philippine inflation variance show that from 1970 onwards, inflation was highly volatile after relatively quiet periods in the 50s and 60s. A closer examination of the results show that from 1970 onwards, periods of high inflation were succeeded by periods of greater uncertainty. This is manifested by the relatively higher variance estimates two to three years after high inflation years. This result presents no evidence of high inflation variance during high inflation periods.

The empirical results of this paper on the sources of inflation confirm the views of most academic economists in the Philippines of the significance of the exchange rate in influencing the inflation process. The exchange rate depreciation explains a larger proportion of the error variance than does money or income growth. The findings of this paper support the BOF view more than the fiscal view of inflation. From these results however, one cannot say that the fiscal aspect of inflation is unimportant.
APPENDIX A
ARCH Estimation

The estimate of the variance of inflation presented in the text makes use of an ARCH model (Engle, 1982). This econometric procedure avoids the assumption of constant unconditional variance of ordinary least squares (OLS) regression and makes a distinction between the conditional and unconditional variance.

The simplest formulation of an ARCH model expresses the conditional variance as a linear function of past squared residuals as follows:

A1) $y_t | \Omega_{t-1} \sim N(x_t \beta, h_t)$

A2) $h_t = a_0 + a_1 e_{t-1}^2 + a_2 e_{t-2}^2 + \ldots + a_p e_{t-p}^2$

A3) $e_t = y_t - x_t \beta$

where $y_t$ is the dependent variable, $x_t$ is a vector of explanatory variables included in the information set $\Omega_{t-1}$, $e_t$ is the disturbance term of the regression (A1), $h_t$ is the conditional variance expressed in equation (A2) and $p$ is the order of the ARCH process. The ARCH regression estimates the parameter vectors $\alpha$ and $\beta$ using maximum likelihood techniques outlined in Engle (1982).

The determination of ARCH effects uses a heteroscedasticity test developed by Breusch and Pagan. The procedure is based on the Lagrange Multiplier (LM) test on the null hypothesis:

$$a_1 = \ldots = a_p = 0.$$
It was shown by Engle that the LM statistic can be calculated by computing the OLS residuals $e_t$, squaring them ($e_t^2$) and regressing on its past values:

$$e_t^2 = \hat{\alpha}_0 + \hat{\alpha}_1 e_{t-1}^2 + \ldots + \hat{\alpha}_p e_{t-p}^2$$

The LM statistic is equal to the $R^2$ of this regression multiplied by the sample size and is distributed as Chi-square with $p$ degrees of freedom. (See Maddala, 1990)
APPENDIX B

Variance Decomposition using VAR

Vector Autoregression (VAR) models may be used as an alternative forecasting method to structural macroeconometric models. They have however been used extensively in testing for exogeneity by examining either the impulse response functions or the decomposition of the forecast error variances of variables. The latter is discussed in this Appendix.¹

A VAR system may be described by the following set of equations:

B1) \( q_t = A(L)q_{t-1} + u_t \); \( A(L) = A_0 + A_1 L + A_2 L^2 + \ldots \)

\[ E(u_t) = 0 \]

\[ E(u_tu_s') = 0 \); \( t \neq s \]

\[ E(u_tu_s') = \Sigma \]

\[ E(q_tu_s') = 0 \); \( t < s \]

where \( q_t \) is an nx1 vector of variables, \( u_t \) is the nx1 error vector. \( L \) is a lag operator and has the property that \( L^k x_t = x_{t-k} \). A VAR system may be viewed as a reduced form version of some structural macroeconometric system.

Assuming the Wold decomposition theorem applies, the vector moving average (VMA) representation of (B1) can be written as

¹Examples of VAR applications are Sims (1980) and Montiel (1989), among others. For a discussion of VAR methods, see Hakkio and Morris (1984).
E2) \( q_t = B^*(L)e_t \); \( B^*(L) = (I - A(L)L)^{-1} \)

where \( B^* \) is a rational polynomial matrix.

The VAR procedure transforms the system such that the contemporaneous covariance matrix is normalized to the identity matrix. The transformation makes use of the Choleski matrix, \( M \), which is lower triangular and \( \Sigma = M^{-1}M^{-1} \) (\( M^{-1} \) is also lower triangular). Thus the system can be written as:

B3) \( Mq_t = MA(L)q_{t-1} + e_t \)

\( e_t = Mu_t \)

\( E(e_t) = E(Mu_t) \)

\( E(e_t e_t') = E(Mu_t u_t' M') = MM = I \)

The equivalent VMA representation of (B3) is:

B4) \( q_t = B(L)e_t \); \( B(L) = B^*(L)M^{-1} \); \( Ee_t = Mu_t \)

\( u_t \) and \( e_t \) are the innovations in \( q_t \) and are equal only if \( M = I \) (Superscripts are variable indices). The difference between the two is due to the information set used to calculate them. Thus for each variable in the system, (B1) and (B3) yields respectively:
\[ u_t^i = q_t^i - E(q_t^i|q_{t-1}^i, q_{t-2}^i, \ldots) \quad \text{for all } i \]

and
\[ e_t^1 = q_t^1 - E(q_t^1|q_{t-1}^1, q_{t-2}^1, \ldots) \]
\[ e_t^2 = q_t^2 - E(q_t^2|q_t^1, q_{t-1}^1, q_{t-2}^1, \ldots) \]

\[ \vdots \]
\[ e_t^n = q_t^n - E(q_t^n|q_t^1, q_t^2, \ldots, q_{t-1}^n, q_{t-1}^1, q_{t-2}^2, \ldots) \]

Notice that \( u_t \) is the prediction error using only past values of \( q_t \) while \( e_t^i \) in addition, includes current values of the \( j \) variables for \( j < i \). For \( u_t \), the ordering of the variables in the vector \( q \) does not matter. On the other hand, \( e_t \) is not invariant to the ordering of the system because of the transformation using the Choleski decomposition. Different orderings generate different \( H_t \) and therefore different information sets for each of the element of the vector \( e_t \).

To look at the decomposition of a one step forecast error variance of a variable, the \( i \)th row of \( (B_t) \) at time \( t+1 \) is
\[ q_{t+1}^i = \sum_{j=1}^{n} \sum_{s=0}^{\infty} b_{ij} e_{t+1-s} \]

and the conditional expectation is
\[ E_t^i q_{t+1}^i = \sum_{j=1}^{n} \sum_{s=1}^{\infty} b_{ij} e_{t+1-s} \]
The forecast error is then:
\[ (q_{t+1}^i - E_t^i q_{t+1}^i) = \sum_{j=1}^{n} b_{ij} e_{t+1} \]

After noting that \( E(e_t e_t^\top) = I \), the one step ahead forecast error variance is calculated:
\[ E_t (q_{t+1}^i - E_t q_{t+1}^i)^2 = \sum_{j=1}^{n} (b_{ij})^2 \]

Each term of the summation on the right hand side \((b_{ij})^2\) is the contribution of the innovations in variable \(j\) on the one step ahead squared prediction error in the variable \(i\). The one step forecast error variance (in percent) in the \(i\)th variable due to an innovation in variable \(j\) is therefore:

\[ D_{t+1}^{ij} = \frac{(b_{ij})^2}{\sum_{j=1}^{n} (b_{0j})^2} \]

where \(D_{t+1}^{ij} = 1\) and \(\sum_{j=1}^{n} D_{t+1}^{ij} = 1\).

The result can be generalized for a \(k\)-step ahead forecast:

\[ D_{t+k}^{ij} = \frac{\sum_{s=0}^{k-1} (b_{ij})^2}{\sum_{j=1}^{n} \sum_{s=0}^{k-1} (b_{sj})^2} \]

and \(\sum_{j=1}^{n} D_{t+k}^{ij} = 1\).

Table 2 in the text shows the computed \(D_s\) only for the inflation rate based on different orderings of the variables in the system and for a ten year ahead forecast.
References


