Solving General Equilibrium Systems
Using SCROP60

by

Carlos C. Bautista

The paper serves to place the software (SCROP60) in the public domain. A guide on how to operate it and its hardware requirements is discussed. A critical general equilibrium example using the software is demonstrated to analyze (1) changes in initial endowments and (2) a corporate income tax problem and the attendant welfare issues involved.

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ABSTRACT

The paper serves to place the software (SCROP60) in the public domain. A guide on how to operate it and its hardware requirements is discussed. A neoclassical general equilibrium example using the software is then presented to analyze (1) changes in initial endowments and (2) a corporate income tax problem and the attendant welfare issues involved.

The software is still crude and quite cumbersome to use, rather slow when the system gets larger but is quite effective especially when the equations get to be highly nonlinear. It does not limit to economic problems; it can handle any simultaneous nonlinear system of equations and may be useful to other sciences. The illustration given in this paper is however an economic example.

The author is a Lecturer at the U.P. School of Economics. The comments of Dr. S. Mohsin are gratefully acknowledged.
Solving General Equilibrium Systems
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Carlos C. Bautista*

I. Introduction

This paper's purpose is twofold. First, it places the software used by the author in his dissertation in the public domain. Second, it demonstrates by way of an example, its use in applied general equilibrium (AGE) analysis. The software contains the algorithm constructed by Powell (1970) for use in mainframe computers. The software developed modifies the input-output system for the mainframe to suit the requirements of a micro computer.

The software is still crude and quite cumbersome to use, rather slow when the system gets large, but is quite effective especially when the equations get to be highly nonlinear. It is not limited to economic problems. It can handle any simultaneous nonlinear system of equations and may be useful to other sciences. The illustration given in this paper is however an economic example.

*The author is a Lecturer at the U.P. School of Economics. The comments of Manuel Montes are gratefully acknowledged.
The second section briefly describes the software and its implementation in a micro computer. The last section gives an example of how a full neoclassical AGE model of a hypothetical closed economy is solved using this software. Parameterization and calibration procedures are also discussed briefly in this section. A trivial experiment on labor supply increases is conducted. A tax experiment is then used to demonstrate how welfare issues can be analyzed using AGE models.

II. The Software

Powell's algorithm was implemented using MS FORTRAN and was written in single precision arithmetic. The main program including a matrix inversion subroutine is contained in the object file named Scrop60.obj.

To use the program, a subroutine containing the system of equations written in FORTRAN source code must be created as a separate text file. This subroutine, when called by the main program, returns the values of all endogenous variables and the residuals of all equations.

The algorithm is said to have found a solution if the sum of squared residuals are less than the user-specified tolerance limit. The subroutine must be named EQNS and must follow all of

1This is simply an expansion of the Edgeworth box.
FORTRAN's syntax. This source file containing EQNS must then be compiled and linked with the main object file (SCROP60). Table 4 shows an example of how the equations are written. Endogenous variables are the FORTRAN variables x while the residuals are labelled f indexed up to the number of equations.

Having done this, the executable file can be called. The program runs into interactive mode and asks a series of questions. It sequentially asks for (1) the number of equations, (2) the number of iterations desired, (3) initial step length to approximate the first derivative, (4) the tolerance limit, (5) the initial values of each endogenous variable, (6) whether to suppress intermediate results from appearing in the console and finally (7) the output filename. The results of the program are written in a text file which can be picked up by a spreadsheet or a word-processor.

Some items above need more explanation. For a system of non-linear equations, \( f(x) = 0 \), the Powell algorithm solves this set of equations using Newton's method (item 3 is used in this regard), calculates the sum of squared residuals:

\[
F(x) = f'f = \Sigma_n[f_1(x)]^2
\]

and tries to achieve an improvement:
\[ F(x_{t+1}) < F(x_t) \]

where the superscript denotes the \( t \)th iteration. The algorithm terminates if \( F(x) \) is less than or equal to the user specified tolerance limit (item 4).\(^2\)

The software requires an IBM PC/XT/AT or compatible with at least one disk drive and 256 kilobytes of memory. The software can accommodate up to 60 equations. A math co-processor significantly increases computation speed but is not required.

III. An Example

The example in this paper is a simple full neoclassical AGE model which is used to conduct two experiments. AGE models are the product of developments in the field of development planning. Essentially these models are supposed to aid planners in the design of policies. To operate an AGE model, one needs a solution algorithm and an equilibrium data set, preferably a social accounting matrix (SAM).\(^3\) In AGE analysis, the SAM is the base by which all policy experiments or simulation results are compared. One may think of the base SAM as an initial equilibrium position. A disturbance in a model which is built

\(^2\)A rigorous discussion of the tests performed can be found in Powell (1970).

\(^3\)In the absence of a SAM, the data set has to be constructed and consistency checks must be performed.
around the base SAM gives rise to a new equilibrium situation—
or a new SAM.

A SAM is a way of organizing a set of economic accounts that
makes it easy for the user to view the intersectoral flows of an
economy. The hypothetical SAM used in this paper is given in
Table 1 and is adopted from Drud (1987). As can be seen, it is a
two-sector, two-factor, two-household economy. Columns indicate
payments or expenditures of economic agents while the rows
represent their revenues.

The circular flow of income is readily discernible in this
Table. For example, two commodities, \( X_1 \) and \( X_2 \) (columns 5 and
6), are produced using capital and labor. Households, \( C_1 \) and \( C_2 \),
receive income from wages and profits in rows 3 and 4 which they
spend on the two commodities. The interrelationships of four
markets, labor, capital and two commodities are effectively
described by the SAM.

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>K</th>
<th>C1</th>
<th>C2</th>
<th>X1</th>
<th>X2</th>
<th>Total</th>
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<tr>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
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<td></td>
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<td>85</td>
<td></td>
<td></td>
<td>160</td>
</tr>
<tr>
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<td>50</td>
<td>50</td>
<td>40</td>
<td>30</td>
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<td>X1</td>
<td>60</td>
<td>65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>125</td>
</tr>
<tr>
<td>X2</td>
<td>60</td>
<td>85</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>145</td>
</tr>
<tr>
<td>Total</td>
<td>160</td>
<td>110</td>
<td>120</td>
<td>150</td>
<td>125</td>
<td>145</td>
<td></td>
</tr>
</tbody>
</table>

\(^4\)See B. King (1978) for an elementary exposition of a SAM.
With the data and the algorithm, one can begin specifying the structure of the model. In practice, the nature of the data and the objective of the modelling exercise governs the manner by which the model is formulated.

The specification of the model is shown in Table 2. The model assumes a perfectly competitive economy where all resources are fully employed. Furthermore, these factors are perfectly mobile so that factor prices are equalized across sectors when an equilibrium position is disturbed.

The production of the two sectors are modeled as constant returns CES functions (equations 2.1 and 2.2). The corresponding factor demand functions derived from optimization processes are given by equations 2.3 to 2.6. Factor market equilibria are shown by equations 2.7 and 2.8. The \( \phi_i \)'s are the substitution parameters equal to \( (1-\sigma_i)/\sigma_i \). \( \sigma_i \) are the substitution elasticities. The \( \delta_i \)'s and the \( \psi_i \)'s are the distribution and scale parameters respectively.

The consumption subsystem (equations 2.11 to 2.14) is the linear expenditure system (LES) which assumes that individuals maximize Stone-Geary utility functions subject to their budget constraints (equations 2.9 and 2.10). The \( \alpha \)'s are the shares of
the household incomes from owning factors of production. The goods market equilibrium conditions are written in equations 2.15 and 2.16. The $\mu_{ijs}$ are the marginal budget shares and the $\theta_{ijs}$ are the base consumption levels of household $i$ of commodity $j$. There are 16 equations and 16 unknowns in the model.

By Walras law however, the equation system is not independent and one equation can be dropped and replaced by a normalization rule to complete the system. The last equation fixes the absolute price level (CPI). In some models, this is chosen as a numeraire.\footnote{In the first experiment on labor supply, all prices are tried out to demonstrate that the choice of the numeraire is arbitrary.}

The system is a standard Walrasian model where only relative prices influence economic behavior. In closed economy neoclassical models of this type, Walras' law holds if three conditions are fully met:

(1) all (factor and commodity) markets are in equilibrium,

(2) all household incomes are spent so that strict equality holds in equations 2.9 and 2.10,
(3) the zero profit condition holds in all industries.\textsuperscript{6}

For the latter, this simply means that for each industry, total revenue equals total cost, i.e.,

\[ P_1 X_1 = P_L L_1 + P_K K_1 \]
\[ P_2 X_2 = P_L L_2 + P_K K_2 \]

These two equations together with equations 2.7, 2.8, 2.15 and 2.16 determine the prices \( P_1, P_2, P_L, P_K \) and the quantities \( X_1 \) and \( X_2 \).

\textsuperscript{6}For an open economy, zero BOP is a condition that must be satisfied. In Clarete and Roumasset (1987), rents due to an assumed fixed factor. At equilibrium, anticipated rent equals actual rent is another condition that must be satisfied.
Table 2
A Neoclassical Model

\[ X_1 = \frac{\Phi_1}{\Phi_1} \left[ \delta_1 L_1 + (1 - \delta_1) K_1 \right] \]

\[ X_2 = \frac{\Phi_2}{\Phi_2} \left[ \delta_2 L_2 + (1 - \delta_2) K_2 \right] \]

\[ L_1 = \frac{X_1}{\Phi_1} \left\{ (1 - \delta_1) \left[ \frac{\delta_1 P_k}{(1 - \delta_1) P_L} \right] \frac{\pi_1}{(1 + \pi_1)} + 1/\pi_1 \right\} \]

\[ L_2 = \frac{X_2}{\Phi_2} \left\{ (1 - \delta_2) \left[ \frac{\delta_2 P_k}{(1 - \delta_2) P_L} \right] \frac{\pi_2}{(1 + \pi_2)} + 1/\pi_2 \right\} \]

\[ K_1 = \frac{X_1}{\Phi_1} \left\{ (1 - \delta_1) P_L \left[ \frac{\pi_1}{(1 + \pi_1)} + 1/\pi_1 \right) \right\} \]

\[ K_2 = \frac{X_2}{\Phi_2} \left\{ (1 - \delta_2) P_L \left[ \frac{\pi_2}{(1 + \pi_2)} + 1/\pi_2 \right) \right\} \]

\[ K^* = K_1 + K_2 \]

\[ L^* = L_1 + L_2 \]

\[ Y_1 = \alpha_{K1} P_k K^* + \alpha_{L1} P_l L^* \geq P_1 C_{11} + P_2 C_{12} \]

\[ Y_2 = \alpha_{K2} P_k K^* + \alpha_{L2} P_l L^* \geq P_1 C_{21} + P_2 C_{22} \]

\[ C_{1j} = a_{i,j} + \frac{\mu_{i,j}}{P_j} (Y_i - \Sigma (\theta_{i,j} P_j)) \]

\[ X_1 = C_{11} + C_{12} \]

\[ X_2 = C_{12} + C_{22} \]

\[ CPI = \Phi_1 \Omega P_1 \]
The numerical implementation of the model involves a process called calibration. Given the specified behavior of economic agents as reflected by the equations and given the data at hand, the parameters of the model can be determined. If the parameters are correctly set, the model should be able to reproduce the data used in the calibration process.

The parameters of this model need to be determined before the base SAM is replicated and a simulation is conducted. In the calibration process, it has been the convention to set all prices equal to one.\(^7\) This practice began with Harberger's (1966) general equilibrium model of taxation. This convention implies that the base SAM gives both the nominal and real figures. The main parameters of the model are shown in Table 3.

\[\begin{align*}
\phi_1 &= 1.966615 \\
\phi_2 &= 1.929015 \\
\delta_1 &= 0.583677 \\
\delta_2 &= 0.704905 \\
\theta_{11} &= 47.4 \\
\theta_{12} &= 12.6 \\
\theta_{21} &= 53.0 \\
\theta_{22} &= 22.0
\end{align*}\]

\[\begin{align*}
\mu_1 &= 0.21 \\
\mu_2 &= 0.16
\end{align*}\]

\[^7\text{A good reference on calibration techniques is Mansur and Whalley (1984).}\]
For the consumption parameters, the marginal budget shares \( \mu_{11} \) and \( \mu_{31} \) were taken from Bautista (1987). The Engel aggregation condition, \( \Sigma \mu_{ij} = 1 \), was used to derive \( \mu_{12} \) and \( \mu_{22} \). The \( \theta_{ij} \)'s were derived using the \( \mu_{ij} \)'s, the average budget shares taken directly from the data and an assumed Frisch parameter.

On the production side two extraneous parameters, the substitution elasticities, were needed to derive the rest of the parameters. The elasticities were lifted from Habito (1984). It is easy to show that from the optimization process, one can get the distribution parameters as (subscripts are omitted):

\[
\delta = \frac{1}{\{(P_K/P_L)(K/L)^{1+\sigma} + 1\}}
\]

Having determined the numerical value of \( \delta \) using SAM data for \( K \) and \( L \) and setting base prices equal to one, this and the zero profit condition is then used to get \( \phi \):

\[
\phi = \frac{(P_KK + PLL)}{P[6L^{-\sigma} + (1 - \delta)K^{-\sigma}]^{-1/2}}
\]

Since the model parameters were derived from the equilibrium data set, the model must be able to replicate the SAM as the base case. In some models which get to be complicated because of its
size, adjustments are made in some parameters (usually the scale parameters) so that a replication is made possible.

Table 4 shows the baserun FORTRAN source file of the model discussed above. The file contains the equations, the conditions that must be met to achieve a Walrasian equilibrium and the parameters computed using the model specification and the base SAM. Notice that in the source code, only the parameters and the fixed factor supplies are needed to replicate the SAM. It is also important to note that a statement of Walras' law must be included in the file.

Column 1 of Table 5 shows the baserun of the model and the results of the first experiment. This column indicates that the model was able to replicate the data from the SAM. This is equivalent to saying that the numerical solution of the model at the base (i.e., without any change in exogenous variables and parameters) is the base SAM. This is necessary because in AGE analysis, the initial data set is taken as the initial equilibrium position.

In this exercise, total labor supply was assumed to increase by 5 percent and real labor income distributed proportionally to the household groups. All the prices were tried out as the numeraire (columns 2 to 6) and obviously the same results on the real magnitudes were achieved since the choice of a numeraire
does not matter and is arbitrary. For each column, all prices and nominal magnitudes should be viewed relative to the numeraire. The results of the experiment were also arranged in a constant SAM form.

The results conform with what is predicted by theory. Looking at the second column where the price of commodity 1 is the numeraire, the increase in labor supply led to a decrease in the relative price of labor and an increase in the price of capital which became relatively scarce. Since the price of capital increased, the relative price of the commodity where the production requires more capital intensive techniques also rose.

Since labor became relatively cheap, capital usage declined and labor inputs increased in the more labor intensive industry 1 which also has a higher substitution elasticity. With fixed factor supplies, capital transfers from industry 1 to industry 2. Capital movements to the capital intensive industry increased labor usage leading to an increase in total output.

In the next simulation, a 50% tax on capital income of industry 2 is imposed and the tax revenue is reallocated back to household 1 and 2 in the proportion 30% and 70% respectively. The wage rate is held as a numeraire in this experiment. The

A similar experiment was done by Harberger (1966). Shoven (1976) later showed arithmetic and units-definition errors in Harberger and made recomputations.
reader should bear in mind that factors are assumed perfectly mobile across sectors. At the initial equilibrium, industry 2 is capital intensive. Household 2 who receives the larger part of capital income is richer than household 1 who obtains income mainly from labor efforts. Three runs of this experiment are conducted with differing substitution elasticities and without changing demand parameters. The following are the substitution elasticities for the three runs:

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_1 )</td>
<td>1.2</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>( \sigma_2 )</td>
<td>0.4</td>
<td>0.4</td>
<td>1.2</td>
</tr>
</tbody>
</table>

For all runs, Table 6 shows that as a result of the tax, output of Industry 2 declines while its price goes up. With perfect mobility of factors, capital transfers from industry 2 to industry 1. The overall (equalized) price of capital declines due to the tax. However, the cost of capital in industry 2, i.e., the price of capital gross of the tax increases. This increase is reflected in the price of industry 2 output. With an elastic demand, consumption of industry 2 output declines.

The sensitivity analysis is carried out to show the magnitude of the burden capital bears with the imposition of the tax. From experiment I to II, \( \sigma_1 \) is decreased with \( \sigma_2 \) remaining the same. From experiment II to III, \( \sigma_2 \) is increased holding \( \sigma_1 \)
constant. The share of capital's burden as a result of the tax may be roughly measured as the ratio of the change in the price of capital and the average tax rate:

\[ P_k^0 - P_k^f = b(R/K^*) \]

where \( b \) is the burden and \( R \) is the tax revenue. The share of capital's burden for runs I, II, III are 0.46, 0.94 and 1.47 respectively. Thus, the burden increases when the untaxed sector's elasticity is decreased or when the taxed sector's substitution elasticity is increased - a result also obtained by the studies mentioned above.

At first glance, one may calculate the total tax revenue as 1 \times 60 \times 0.5 = 30. However, if general equilibrium effects are taken into account, the tax revenue is considerably lower (See Table 7a). The resulting distribution of income is also shown in this Table.

AGE models are used in welfare analyses of policies which result in price changes. With the results of a policy simulation, it becomes convenient to generate numerical measures of welfare such as the equivalent variation (EV) and the compensating variation (CV).

9See Atkinson and Stiglitz (1980).
The EV is an evaluation using initial prices as the base and seeks to determine what income change at the new prices will be necessary to bring the consumer to the previous utility level. The CV on the other hand uses the new prices as its base and tries to determine the income change needed to compensate the consumer for the price change. These measurements make use of the expenditure functions, e, of each consumer, i.e.,

\[ EV = e(P, V(P_n, Y_n)) - e(P, V(P, Y)) \]
\[ CV = e(P_n, V(P_n, Y_n)) - e(P_n, V(P, Y)) \]

where the superscript n denotes new levels. To get the expenditure functions, the indirect utility function, V, must first be determined. The utility function from which the LES is obtained is \( U = \pi_1(C_1 - \theta_1)^{\alpha_1} \) and the corresponding indirect utility function is \( V = (Y - \Sigma \theta_1 P_1) \pi_1(\mu_1/P_1)^{\alpha_1} \). Inverting this yields the expenditure function:

\[ e = \Sigma \theta_1 P_1 + V \pi_1(P_1/\mu_1)^{\alpha_1} \]

The redistribution of the tax revenue to the two households show a decline in nominal income of the second group. The computed EV and CV for each household and the welfare change as percent of total income in the tax experiment of this paper are shown in Table 7b. Net welfare losses amounted to less than one percent of income due to the redistribution.
subroutine eqns(n,x,f)
dimension x(1),f(1)

file = Baserun.f

…………………………………………………………………………………………..
Ces parameters: sigma1 = 1.2 delta1 = .583677 phi1 = 1.966615
sigma2 = 0.4 delta2 = .704905 phi2 = 1.929015

Les parameters: mu1 = .21 mu21 = .16

Variables:

x(1) = x1 x(5) = L1 x(9) = Pk x(13) = C11
x(2) = x2 x(6) = L2 x(10) = Pl x(14) = C12
x(3) = P1 x(7) = K1 x(11) = Y1 x(15) = C21
x(4) = P2 x(8) = K2 x(12) = Y2 x(16) = C22

…………………………………………………………………………………………..

Numeraire

f(1) = x(3) - 1.0

Waldas Law

f(2) = x(1)*x(3) - x(3)*(x(13)+x(15)) + x(2)*x(4)
   1 - x(4)*(x(14)+x(16))

Factor Demands - Labor

f(3) = (x(1)/1.966615) * (.416323 * (.583677 * x(9)/(.416323*
   1 x(10))) ** (-2) + .583677) ** (-6.0) - x(5)

f(4) = (x(2)/1.929015) * (.295095 * (.704905 * x(9)/(.295095*
   1 x(10))) ** .6 + .704905) ** (1/1.5) - x(6)

Factor Demands - Capital

f(5) = (x(1)/1.966615) * (.583677 * (.416323 * x(10)/(.583677*
   1 x(9))) ** (-2) + .416323) ** (-6.0) - x(7)

f(6) = (x(2)/1.929015) * (.704905 * (.295095 * x(10)/(.704905*
   1 x(9))) ** .6 + .295095) ** (1/1.5) - x(8)
FACTOR MARKET EQUILIBRIUM

\[ f(7) = x(7) + x(8) - 110 \]
\[ f(8) = x(5) + x(6) - 160 \]

INITIAL ENDOWMENTS

\[ f(9) = x(11) - (x(7)+x(8)) \times x(9) \times 3/11 \]
\[ 1 - (x(5)+x(6)) \times x(10) \times 9/16 \]
\[ f(10) = x(12) - (x(7)+x(8)) \times x(9) \times 8/11 \]
\[ 1 - (x(5)+x(6)) \times x(10) \times 7/16 \]

CONSUMER DEMANDS - LES

\[ f(11) = 47.4 + (x(11) - 47.4 \times x(3) - 12.6 \times x(4)) \times .21/x(3) - x(13) \]
\[ f(12) = 12.6 + (x(11) - 47.4 \times x(3) - 12.6 \times x(4)) \times .79/x(4) - x(14) \]
\[ f(13) = 53.0 + (x(12) - 53.0 \times x(3) - 22.0 \times x(4)) \times .16/x(3) - x(15) \]
\[ f(14) = 22.0 + (x(12) - 53.0 \times x(3) - 22.0 \times x(4)) \times .04/x(4) - x(16) \]

GOODS MARKET EQUILIBRIUM - commodity 2

\[ f(15) = x(2) - x(14) - x(16) \]

ZERO PROFIT CONDITION - industry 1

\[ f(16) = x(1) \times x(3) - (x(5) \times x(10) + x(7) \times x(9)) \]

return
end
Table 5
First Experiment

<table>
<thead>
<tr>
<th>Numeraire =</th>
<th>BASE</th>
<th>P₁</th>
<th>P₂</th>
<th>Pₚ</th>
<th>Pₗ</th>
<th>CPI</th>
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<td>1 X₁</td>
<td>125</td>
<td>127.6544</td>
<td>127.6555</td>
<td>127.6554</td>
<td>127.6540</td>
<td>127.6551</td>
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<tr>
<td>2 X₂</td>
<td>145</td>
<td>150.2418</td>
<td>150.2416</td>
<td>150.2414</td>
<td>150.2369</td>
<td>150.2393</td>
</tr>
<tr>
<td>3 P₁</td>
<td>1</td>
<td>1.0000</td>
<td>0.9472</td>
<td>0.9622</td>
<td>1.0259</td>
<td>0.9709</td>
</tr>
<tr>
<td>4 P₂</td>
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<td>1.0558</td>
<td>1.0000</td>
<td>1.0158</td>
<td>1.0831</td>
<td>1.0251</td>
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<td>5 L₁</td>
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<td>78.9773</td>
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<tr>
<td>6 L₂</td>
<td>85</td>
<td>89.0230</td>
<td>89.0227</td>
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<td>89.0219</td>
</tr>
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<td>48.7517</td>
<td>48.7518</td>
<td>48.7531</td>
<td>48.7524</td>
</tr>
<tr>
<td>8 K₂</td>
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<td>61.2483</td>
<td>61.2482</td>
<td>61.2469</td>
<td>61.2476</td>
</tr>
<tr>
<td>9 Pₚ</td>
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<td>1.0393</td>
<td>0.9844</td>
<td>1.0000</td>
<td>1.0662</td>
<td>1.0091</td>
</tr>
<tr>
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<td>0.9233</td>
<td>0.9379</td>
<td>1.0000</td>
<td>0.9464</td>
</tr>
<tr>
<td>11 Y₁</td>
<td>120</td>
<td>127.9024</td>
<td>121.1472</td>
<td>123.0621</td>
<td>131.2112</td>
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<td>13 C₁₁</td>
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<td>16 C₂₂</td>
<td>85</td>
<td>87.3590</td>
<td>87.3586</td>
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SOLUTION SAM
5.0 percent increase in Labor Supply
Numeraire = P₁

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<th>L</th>
<th>K</th>
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<th>X₁</th>
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<th>Total</th>
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Total 168.0 110 124.5000 153.5000 127.6544 150.2418
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<th>III</th>
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<td>L1</td>
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<td>K1</td>
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<td>55.7361</td>
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<td>Y1</td>
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### Table 7a
Tax Revenue, Nominal Incomes and Transfers

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<tr>
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<td>Transfers - C2</td>
<td>7.3045</td>
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<td>K income - C1</td>
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<tr>
<td>L income - C1</td>
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<td>90.0000</td>
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<tr>
<td>L income - C2</td>
<td>70.0000</td>
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<td>70.0000</td>
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<tr>
<td>Total income - C1</td>
<td>133.9661</td>
<td>129.2822</td>
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<tr>
<td>Total income - C2</td>
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<td>Total income</td>
<td>283.0633</td>
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### Table 7b
Equivalent and Compensating Variations

<table>
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<tr>
<td></td>
<td>EV</td>
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<td>EV</td>
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<tr>
<td>Total</td>
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<td>Loss as % of Y</td>
<td>0.401%</td>
<td>0.470%</td>
<td>0.301%</td>
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References


