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Consumer Choice of Qualities

by

José Encarnación, Jr.

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Abstract

This paper proposes a treatment of quality variations based on a lexicographic preferences assumption. The main result is that the average quality of goods consumed is higher at higher incomes which, though commonly observed, is not explained by standard theory.
1. Introduction

The aim of this paper is to present a new theory of consumer choice where there are quality differences among goods. Section 2 gives a quick review of lexicographic preferences, which provide a more general framework for the analysis of choice by viewing utility as a vector. Section 3 treats a special case that parallels standard theory in assuming strict quasi-concavity for each component of the utility vector. This restriction is relaxed in Section 4, and Section 5 considers the question of qualities and draws some implications under price and income changes. The main result is that the average quality of goods consumed is higher at higher incomes. It is also a consequence that almost always, higher quality goods will be substituted for lower quality ones if any of their prices is lowered, provided of course that they were among the consumer's purchases to begin with.

2. Lexicographic Utility

Let the utility of \( x \), a point in consumption goods space, be written as \( u(x) = (u_1(x), u_2(x), \ldots) \) where the \( i \)-th component corresponds to the \( i \)-th most important criterion or dimension of choice. The underlying idea is the existence of multiple needs and desires which form a hierarchy according to priority or importance. Those belonging to the same level in the hierarchy are subsumed under the same utility component, and \( u_i(x) > u_j(y) \) if \( x \) is preferable to \( y \) on the basis of the \( i \)-th criterion. Like the standard utility function, \( u_i \) is unique up to a positive monotone transformation.

The usual textbook statement of lexicographic preferences is that \( x \) is preferred to \( y \) if the first nonvanishing difference \( u_i(x) - u_i(y) \),
$i = 1, 2, \ldots$, is positive, which we will call L ordering. Most economists are correctly skeptical about the analytical usefulness of L ordering, for if $u_i$ is strictly quasiconcave (as is often assumed about the standard utility function), this suffices to determine choice and therefore the other components of utility beyond the first are redundant. Georgeffu-Roegen (1954)\textsuperscript{1/} has made clear, however, that a more useful representation of the preference relation is different.

Given the function $u_i$, let $w_i$ denote a "satisficing" level\textsuperscript{2/} such that if $u_i(x) \geq w_i$ and $u_i(y) \geq w_i$, then a person would not decide between them according to $u_i$ (say food value) but on the basis of some other dimension (perhaps taste in the literal sense).\textsuperscript{3/} Writing $v_i(x) = \min \{u_i(x), w_i\}$, the basic postulate of what we will call L\textsuperscript{2} ordering is that $x$ is preferred to $y$ if the first nonvanishing difference $v_i(x) - v_i(y)$, $i = 1, 2, \ldots$, is positive. The effective utility, so to speak, thus associated with $x$ is $v(x) = (v_1(x), v_2(x), \ldots)$ instead of the $u(x)$ of L ordering.

Let $p \geq 0$ be the price vector and $Y > 0$ the budget so $S_0 = \{x \geq 0 \mid p^t x \leq Y\}$ is the budget constraint set--it will be understood throughout that $x \geq 0$--and write

$$\theta_i = \max_{x} \{v_i(x) \mid x \in S_i-1\} \quad i = 1, 2, \ldots \quad (1)$$

$$S_i = \{x \in S_i-1 \mid u_i(x) \geq \theta_i\} \quad i = 1, 2, \ldots \quad (2)$$

Then $S^\cmp$, the choice on $S_0$, is given by

$$S^\cmp = S_0 \cap S_1 \cap S_2 \cap \ldots \quad (3)$$

One considers first only those $x$'s whose $u_i(x) \geq \theta_i$ and drops everything
else as a possible choice. If $S_1$ is a many-element set, which is likely if $\theta_1 = u_1^*$, the second step selects only those $x$'s whose $u_2(x) \geq \theta_2$, giving $S_2 \subseteq S_1$, and so on. Every succeeding dimension thus narrows the selection.

For simplicity it will be assumed that given any $x$ there is an $i$ such that $u_i(x) < u_i^*$, ruling out total satiation, and that given any $y \neq x$ there is an $i$ such that $v_i(x) \neq v_i(y)$, ruling out complete indifference, so that $S^*$ is a singleton set. Unless stated otherwise, the indices $k$ and $j$ will be used to denote $\theta_k$ as the first $\theta_i < u_i^*$ in (1) and $S_j$ as the first singleton $S_i$ in (2). Since all $u_i^* (i < k)$ are attainable, we will refer to $u_k$ as the priority objective, or simply the objective, and since one will want to maximize $u_j(x)$ subject to $x \in S_{j-1}$, $u_j$ will be called the maximand.

In the next section, standard results will be placed in a broader setting.

3. A Special Case

It is clear that if $\theta_i < u_i^*$ and the function $u_i$ is strictly quasi-concave, then $S_i$ is a singleton. Suppose each $u_i$ is strictly quasi-concave. Then $k = j$, $\theta_i = u_i^*$ for all $i < j$, and the problem for the consumer is to maximize $u_j(x)$ subject to $p^i x = Y$ and $u_i(x) \geq u_i^*$, $i = 1, \ldots, j-1$. The Kuhn-Tucker conditions below, suppressing the superscript in the optimal $x^0$ except when needed for greater clarity, are necessary and sufficient to solve the problem (Arrow and Enthoven 1961, Theorems 1 and 2):
\[ Y - p'x \geq 0 \]  
(4)

\[ (Y - p'x)\lambda_0 = 0 \]  
(5)

\[ u_i(x) - u_i^* \geq 0 \quad i = 1, \ldots, j-1 \]  
(6)

\[ (u_i(x) - u_i^*)\lambda_i = 0 \quad i = 1, \ldots, j-1 \]  
(7)

\[ U_r - p_r\lambda_0 \leq 0 \quad r = 1, \ldots, n \]  
(8)

\[ (U_r - p_r\lambda_0)x_r = 0 \quad r = 1, \ldots, n \]  
(9)

\[ x \geq 0 \]  
(10)

\[ \lambda \geq 0 \]  
(11)

where \( \lambda = (\lambda_0, \lambda_1, \ldots, \lambda_{j-1}) \) is a vector of Lagrange multipliers, \( u_{ir} = \partial u_i / \partial x_r \), and

\[ U_r = u_{jr} + \sum_{i=1}^{j-1} \lambda_i u_{ir} \]  
(12)

Some implications are straightforward. Since \( U_r > 0 \) for some \( r \) (otherwise the consumer would be at a point of complete satiety), \( \lambda_0 > 0 \) from (8) so \( Y - p'x = 0 \) in (4) because of (5). From (8) and (3),

\[ x_r = 0 \text{ if } U_r/p_r < \lambda_0 \]  
(13)

\[ U_r/p_r = \lambda_0 \text{ if } x_r > 0 \]  
(14)

and therefore

\[ U_r/U_s = p_r/p_s \text{ if } x_r, x_s > 0 \]  
(15)

which have simpler analogues in standard theory. With utility as a vector, \( \lambda_0 \) is the "marginal utility" of income. Looking at (4)-(12), multiplication of \((p, Y)\) by a positive scalar will change \( \lambda_0 \) but not \( x^0 \); i.e. the demand function \( f, x^0 = f(p, Y) \), is homogeneous of degree zero.
In order to see the effects of variations in $p$ and $Y$ similar to familiar ones, consider the case where such variations leave the sets $J = \{i| \lambda_i > 0\}$ and $K = \{r| x_r > 0\}$ unchanged. As Appendix A shows, one obtains the Slutsky-type equation

$$\frac{\partial x_s}{\partial p_r} = \frac{\partial x_s}{\partial p_r} \bigg|_{Y = \text{const}} = - \frac{\partial x_s}{\partial Y}$$ (16)

where $V$ is a vector whose components are $u_i (i \in J)$ and $u_j$. From McKenzie (1957) or Hurwicz and Uzawa (1971, Theorem 1), which do not require the preference ordering to be representable by a real valued utility function, the matrix of substitution terms given by (16) is symmetric and negative semidefinite.

Observing that nothing in Section 2 precludes $u_j$ being a function of $u_1, \ldots, u_{j-1}$, thus permitting tradeoffs among different criteria within the component $u_j$, the standard theory of consumer choice can be thought of as the special case where each $u_i$ is strictly quasiconcave and all of (6) are satisfied as strict inequalities. Then, every $\lambda_i = 0$ in (7), (15) reduces to the usual $u_{jr}/u_{js} = p_r/p_s$, and (16) reduces to the standard Slutsky equation where only the value of the maximand $u_j$ is held constant. In effect, the implicit assumption of standard theory is that every $u_i (i < j)$ is surpassed at $x^*$, the standard utility function being some composite $u_j$, which holds in some but evidently not in all cases.

The assumption of strict quasiconcavity rules out linear segments of $u_i$ indifference surfaces, which can be relaxed.
4. A More General Case

Since a particular want can be accommodated by some goods but not by others, corresponding to each \( u_i \) let

\[ G_i = \{ r \mid \forall x: u_i(x) \geq u_i^r + u_i^r > 0 \} \]

denoting goods by their indices. If \( r \) does not belong to \( G_i \), then \( u_i^r = 0 \) identically (e.g. no amount of clothing does anything to fill the need for food). Thus from (14), \( x_r > 0 \) only if \( r \in G_i^c \) for some \( i \neq j \). A good will not be present in the consumer's choice unless it is a \( G_i \) good, \( i \neq j \).

Consider two \( G_i \) goods \( r \) and \( s \). We will say that they are perfect substitutes with respect to \( u_i \)—they are \( \phi_i \) substitutes for short—if for some constant \( a_{rs} \), \( u_i^r = a_{rs} u_i^s \) everywhere (at every point \( x \)) and therefore the \( u_i \) indifference curves relating \( r \) and \( s \) are parallel straight lines. There is then a set of \( \phi_i \) substitutes, possibly empty, associated with each \( u_i \). If nonempty, \( u_i \) cannot be strictly quasiconcave.

We relax the assumption in Section 3 by now assuming that each \( u_i \) is quasiconcave but satisfies the following:

**Condition \( \phi \).** If there are \( \phi_i \) substitutes and \( x(i) \) is the corresponding subvector of \( x \), then, given any constant \( \bar{x}(i) \), \( u_i \) is strictly quasiconcave on the set \( \{ x \mid x(i) = \mu \bar{x}(i), \mu > 0 \} \). If there are no \( \phi_i \) substitutes, \( u_i \) is strictly quasiconcave.

Thus if the \( \phi_i \) substitutes are held to fixed proportions—this includes the case of zero quantities for all except one of them—then strict quasi-
concavity holds on the set of \( x \)'s so restricted, which does not seem unreasonable.

Two cases will need to be distinguished in the maximization problem when we consider the implications of a larger \( S_a \) (due to \( dY > 0 \), or to \( dp_r < 0 \) provided \( x_r > 0 \)): Case I, where \( S_k \) is a singleton; and Case II, where it is not, and therefore \( k < j \). It follows from Condition \( \psi \) that \( S_k \) is a many-element set in Case II only because of some \( \phi_k \) substitutes which, at the given prices, permit \( u_k \) to be maximized at more than one point \( x \in S_{k-1} \). (Suppose \( r \) and \( s \) are \( \phi_k \) substitutes and \( u_{kr}/p_r = u_{ks}/p_s \). Then \( du_k = u_{kr}dx_r + u_{ks}dx_s = 0 \) for all \( dx_r \) and \( dx_s \) satisfying \( dY = p_r dx_r + p_s dx_s = 0 \), and \( S_k \) cannot be a singleton.) We will say that these \( \phi_k \) substitutes "form" \( S_k \)--they make it a many-element set--or that these goods "generate" a maximand different from the objective. Clearly, some of them must also be \( S_j \) goods, or \( u_j \) could not pick out the singleton \( S_j \).

Noting that in Case II, \( u_k(x) \geq \theta_k \) is a binding constraint in the problem, \( S_k \) is a subset of the budget (hyper)plane \( \{ x \mid p' x = Y \} \). Consequently a parallel shift of the budget plane due to \( dY > 0 \) gives a new \( S_k \) which is a many-element set again, because of the parallel \( u_k \) indifference surfaces of \( \phi_k \) substitutes. The objective is unchanged by \( dY > 0 \), since \( \theta_k < u_k^* \) to begin with. A large enough increase in \( Y \) will, however, make some utility component with a higher index become the objective. As one might expect, this will involve qualitative changes.
5. Quality Variations

In the classic formulations of Theil (1952) and Houthakker (1952), a commodity is a set of related goods differing only in quality level. Judgment of quality is that of the consumer, who can always ignore a higher priced good unless its quality is also higher. Quality being included as a separate argument in a real valued utility function, a commodity is represented by two dimensions: its quantity—all the goods belonging to a commodity are measured in the same physical units—and its quality level. This approach is unnecessarily restrictive however. First, it requires a complete ordering of the goods belonging to a commodity, which is often not possible. Second, the consumer cannot choose to have two or more qualities of the same commodity (e.g. butter and margarine). If the consumer does so, Theil considers only their average quality while Houthakker treats them as belonging to different commodities. Third, the implication is that the consumer would always be willing to give up some quality for a sufficiently larger quantity, which seems contrary to fact. Unless a person is very poor, he may well choose not to sacrifice any quality for more quantity (as in the case of food, or children). More important, it is left unexplained why quality levels may be expected to rise with income, which is the empirical observation. In order to have an explanation, we have to look at "the reasons why consumers want certain goods" (Houthakker 1952, p. 163).

Lancaster (1971, p. 6) goes in the right direction by arguing that it is the "characteristics" of goods, "those objective properties that are relevant to choice", that people want. In his approach there is a mapping from consumption goods space to characteristics space, and defining the utility function on the latter, preferences over goods are derived from
preferences over characteristics. Lancaster considers a possible correspondence between wants and characteristics, hence hierarchical and satisfaction possibilities, but does not pursue the matter. As in standard theory, his utility function is real valued and limited in scope. In the approach that we will follow, characteristics are bypassed and the dimensions of utility are directly the reasons for choice.

Since any particular good \( r \) does not contribute towards satisfying more than a relatively few wants, \( u_{ir} = 0 \) identically for some \( i \). In addition to the physical properties that goods belonging to a commodity \( C \) must have, \( C \) can be described by a set of indices \( I(C) \) such that \( C = \bigcap_{i \in I(C)} G_i \). Accordingly, what the goods belonging to \( C \) have in common is their value for some components of utility and their irrelevance elsewhere: if \( r \in C \), then \( u_{ir} > 0 \) only if \( i \in I(C) \).

Denote by \( a(C) \) the good belonging to \( C \) whose quality level is \( \alpha \). For convenience simply write \( \alpha = a(C) \), \( \beta = b(C) \), etc., which will be referred to as \( C \) goods. We propose to say that

\[ aQ_b \quad (\alpha \text{ has higher quality than } \beta) \text{ if } u_{i\alpha} \geq u_{i\beta} \text{ for all } i \quad \text{and} \quad u_{h\alpha} > u_{h\beta} \text{ for some } h, \quad (17) \]

\[ aQ_{h \beta} \quad (\alpha \text{ has } u_{h} \text{ higher quality than } \beta) \text{ if } aQ_b \text{ and } u_{h\alpha} > u_{h\beta}, \quad (18) \]

\[ aR_{h \beta} \quad (\alpha \text{ is } u_{h} \text{ better than } \beta) \text{ if } aQ_{h \beta} \text{ and } u_{h \alpha} > u_{h \beta} \text{ / } p_{\alpha} > u_{h \beta} / p_{\beta}, \quad (19) \]

\[ aR_{h \beta} \quad (\alpha \text{ is } u_{h} \text{ superior to } \beta) \text{ if } aR_{h \beta} \text{ and for all } \beta' \neq \alpha, \quad aR_{h \beta'} \text{ if } \beta' R_{h \beta}, \quad (20) \]
the inequalities holding everywhere. $Q$ is transitive and a partial ordering of $C$ goods, in accordance with the fact that in some cases neither $\alpha$ nor $\beta$ can be said to have higher quality than the other. If $\alpha R_{\beta}$ we can take it that $p_{\alpha} > p_{\beta}$ for otherwise, $\beta$ could not be considered for purchase. Definition (19) seems in conformity with common usage: one product is said to be a better buy than another, even though it costs more, if it gives greater value for the price in some sense. Notice that if $\alpha R_{\beta}$, it may be that $\alpha$ and $\beta$ are $\phi_i$ substitutes for some $i \in I(C)$, but if $\alpha$ and $\beta$ form $S_k$ in Case II, then $u_{k\alpha}/p_{\alpha} = u_{k\beta}/p_{\beta}$ and one does not have $\alpha R_{\beta}$. Unless stated otherwise, it will be understood that quality comparisons pertain to the maximand $u_j$, which is the utility component that determines the optimal choice.

Let $x_C$ be the vector of $C$ goods quantities. We will say that the commodity $C$ is $u_k$ normal—recall that the priority objective is $u_k$—if given $dY > 0$ and constant prices, more is spent on it, and if given $dp_r < 0$ (provided $x_r > 0$) and the same income, the expenditure on $C$ is greater than the cost of the old $x_C$ at the new prices. Clearly, at least one $C$ is $u_k$ normal. Unless stated otherwise, a $u_k$ normal commodity will be understood when we speak of a normal $C$.

Income changes

Let $dY > 0$ and assume the following hypothesis, which will be referred to as $H$: $C$ is normal, $x_\beta > 0$, and $\alpha R_{\beta}$ (i.e., $\beta$ of a normal commodity is present in the choice but there is a superior good $\alpha$, possibly absent before the change in income). Then,

$$u_{j\alpha}/p_{\alpha} > u_{j\beta}/p_{\beta}. \quad (21)$$
We will consider the effects of a larger outlay on \( C \) in the two possible cases.

Case I. With the \( u_i^j (i < j) \) constraints satisfied, one would be more interested in \( \alpha \) because of (21). An extra dollar spent on \( \alpha \) makes \( u_j \) larger by \( u_{ja}/p_\alpha \). On the other hand, if the consumer substitutes \( \alpha \) for \( \beta \) by buying one unit more of \( \alpha \) and \( b = \min \{ u_{ia}/ u_{ig} > 0 \mid i < j \} \) units less of \( \beta \), expenditure on \( C \) is increased by \( p_\alpha - b p_\beta \) but \( u_j \) is raised by \( u_{ja} - bu_{j\beta} \) without violating the \( u_i^j (i < j) \) constraints. From (21) it follows directly that

\[
(u_{ja} - bu_{j\beta}) > (p_\alpha - b p_\beta) \frac{u_{ja}}{p_\alpha}.
\]  

(22)

Thus if \( p_\alpha > b p_\beta \)

\[
\frac{(u_{ja} - bu_{j\beta})/(p_\alpha - b p_\beta)}{u_{ja}/p_\alpha} \geq 1
\]

and therefore the substitution gives a higher \( u_j \) per dollar. If \( p_\alpha = b p_\beta \) hence \( u_{ja} > bu_{j\beta} \), the result is the same. If \( p_\alpha < b p_\beta \), which means there is even some "savings" the inequality in (23) is reversed: any reduction in the \( u_j \) level due to the substitution can be more than made up by using the savings to buy more \( \alpha \). In short, an optimal allocation implies less \( \beta \) and more \( \alpha \) than before. (Note that the argument does not require \( x_\alpha > 0 \) in the choice before or optimality of the latter at the then existing \( p \) and \( Y \); cf. Appendix B.)

Case II. \( \delta \) is automatically raised by the increase in \( Y \). As noted in Section 4 however, the corresponding \( S_\delta \) is still a many-element set hence the maximand, which determines the optimal allocation, remains the same. The result is therefore the same, and we have
Proposition 1. Given $H$, a larger income implies a substitution of the higher quality good $a$ for the lower quality $b$, hence an increase in the average quality consumed.

The reason for buying $b$ to begin with is its greater $u_i$ value per dollar's worth for some $i < k$, or $b$ would have never appeared in the choice. It is even possible that $u_{ib}/p_b > u_{ia}/p_a$ for all $i < k$, $i \in I(C)$, in which case $a$ must be absent from the choice at lower incomes where the objective has a lower index. This leads us to the next point.

If $Y$ increases enough for $u_k$ to reach $u_k^*$, the objective will be a new $u_{k'}$, $k' > k$. (It need not be $u_{k+1}$, for $u_{k+1}$ may have been attained in the course of reaching some $u_i^*$, $i \leq k$.) Consider the set of $G_k$ goods: some of them are in the present choice, or else none. If the latter, at least one will be bought at a sufficiently higher income $Y'$ where $u_k$ is the objective. If the former, suppose that a $u_{k'}$ better good is absent from the present choice. It is clear that some such good would then be bought at $Y'$, giving

Proposition 2. At a sufficiently higher income, a "new" good will be purchased unless no better good is available, and if any is available, at least one will enter the choice.

Suppose income rises from $Y$ to $Y''$ and a higher quality good—the particular utility component $u_h$ is unspecified as in (17)—enters the choice. There is no reason to have a higher value of any $u_i$ with $\theta_i = u_i$, so the good must be of a higher quality as regards some $u_h$ with $h > k$. If that good is $u_k$ better at $Y''$, certainly it is also $u_k$ better at $Y$ since $R_k$ is independent of income, in which case it should have been
bought earlier. Therefore \( h > k \), and it is bought now only because \( u^* \)
has been reached and there is a new objective with a higher index hence less
important. Thus

Proposition 3. Prices being constant, the entry of a higher quality
good in the choice at a higher income implies a less important objective.

This may have possibilities for welfare comparisons but our present
interest lies in Propositions 1 and 2. As income rises to higher levels,
new goods appear in the consumer's choice and higher quality goods displace
inferior ones.\(^5\) We would thus expect the average quality of goods consumed
to be higher at higher incomes. However, this empirical fact does not result
only from one-one displacements, so to speak, where for each \( \beta \) there is a
higher quality \( \alpha \). The more general case involves displacements where a
group of goods is substituted for another group.

Let \( A \) and \( B \) be subsets of \( C \)--\( A \) and \( B \) are not necessarily
mutually exclusive--and write \( u_{iA} = \sum_{r \in A} u_{ir} \) and \( p_A = \sum_{r \in A} p_r \). In (17)-(20), put \( A \) and \( B \) in place of \( \alpha \) and \( \beta \) throughout (so \( u_{ia} \)
becomes \( u_{iA} \); etc.), and replace \( Q, Q_h, R_h, \) and \( R_h \) by \( Q', Q'_h, R'_h, \) and \( R'_h \),
respectively, to define the latter.

Consider the hypothesis \( H' : C \) is normal, \( x_s > 0 \) for all \( s \in B, \)
and \( A \neq B \). No particular \( r \in A \) need have higher quality than any particular
\( s \in B, \) so \( H' \) is a weaker hypothesis than \( H \). Let us say that \( A \) is
increased by one unit if every \( r \in A \) is increased by one unit--the
quantities \( x_r, r \in A, \) are not necessarily equal--and that \( A \) is substituted
for \( B \) if \( A \) is increased and \( B - A \) is decreased. Exactly the same
reasoning for Proposition 1, replacing \( \alpha \) and \( \beta \) by \( A \) and \( B \) throughout,
then yields the more general

**Proposition 1'.** Given \( H' \), a larger income implies a substitution of the higher quality group of goods \( A \) for the lower quality \( B \), hence an increase in the average quality consumed.

Proposition 1 is only the special case where \( A \) and \( B \) are one-element sets. Another special case, where only \( B \) is a one-element set, explains the common phenomenon of a variety of goods displacing a single good. For example, "If a woman's income is so low that she can afford only one pair of shoes, she will buy a pair that can be used on a great variety of occasions. If she can afford two pairs, she will not buy two of the same kind as before, but two pairs that complement each other by being useful on different occasions". Neither of the two new pairs may have higher quality than the original pair, but as a group they are better and they therefore displace the all-purpose shoes.

Simple changes in the arguments for Propositions 2 and 3 similarly give stronger versions where "available group of goods" and "higher quality group of goods" replace "available good" and "higher quality good", respectively.

**Price changes**

Suppose \( x_\alpha > 0 \) in addition to \( H \), and let \( dp_\alpha < 0 \). Given the new prices, the old \( x_\gamma \) can be purchased with a smaller outlay, and more will be spent on \( C \).

**Case I.** The Case I argument leading to Proposition 1 is fully applicable—the prices there should be read as the new prices—and \( \alpha \) will displace \( \beta \).
Case II. There are two possibilities. (i) If $\alpha$ is not among the $\phi_k$ substitutes that form $S_k$, the fact remains that $S_k$ is a many-element set under the price change, so the allocation is determined by $u_j$ with the same result. (ii) If $\alpha$ is one of the goods which generate a maximand other than the objective, then $\beta$ is also a $G_k$ good. Consider the situation prior to the price change: since $S_k$ is a subset of the budget plane where $u_k$ is maximized, $du_k = u_{k\alpha}dx_{\alpha} + u_{k\beta}dx_{\beta} = 0$ and $dy = p_\alpha dx_{\alpha} + p_\beta dx_{\beta} = 0$ both hold at $x^0$, hence $u_{k\alpha}/p_\alpha = u_{k\beta}/p_\beta$ at $x^0$ whether or not $\alpha$ and $\beta$ are $\phi_k$ substitutes. With the new price $p'_\alpha$, we now have $u_{k\alpha}/p'_\alpha > u_{k\beta}/p'_\beta$. Thus the same argument in Case I, putting $k$ in place of $j$, gives

**Proposition 4.** If $H$ holds, $\alpha$ will displace $\beta$ if $\alpha$ is present in the choice and its price is decreased.

Let $\beta$ have a lower price $p'_\beta$ instead, but maintain $\alpha_\beta$. Case I and Case II(i) of Proposition 4 can be repeated almost word for word, with the result that $\alpha$ will displace $\beta$. However, Case II(ii) is different: with $\beta$ now more attractive at $u_k$ which has a higher priority than $u_j$, using exactly the same reasoning gives the opposite result. (Interchange $\alpha$ and $\beta$ throughout, and put $j = k$ in Case I of Proposition 1.) Thus

**Proposition 5.** Suppose $H$ holds and the price of $\beta$ is decreased but $\alpha$, which is present in the choice, remains better. Then $\alpha$ will displace $\beta$ unless the latter is one of the goods which generate a maximand different from the objective, in which case $\beta$ will be substituted for $\alpha$. 
Such a case rests on the fortuitous circumstance that relative prices are just right in order for a lower-priced \( \beta \) to be one of the goods that form \( S_k \), so we would conclude from Propositions 4 and 5 that \( \alpha \) will displace \( \beta \) almost always under the stated conditions. A similar conclusion applies to groups \( A \) and \( B \) if any price \( p_s \) \((s \in A \cup B)\) is decreased, exceptions arising only where \( s \in B - A \) is one of the goods which form \( S_k \).

6. Concluding Remark

It is generally observed that different income classes buy different goods and that higher income classes consume higher quality goods. This observation is not deducible from the extension of standard theory to quality choice but it follows from the model of this paper, which assumes that the consumer has lexicographic preferences "not because it would constitute a more convenient approach or lead to a simpler schema, but because it offers a more adequate interpretation of the structure of our wants" (Georgeescu-Roegen 1954, p. 519).
Appendix A

Consider the equations in (4), (6) and (8) determined by the solution. Taking total differentials, one has the following system that includes only the results from those equations:

\[
\begin{bmatrix}
W & \mathbf{v}_1 & \ldots & \mathbf{v}_{j-1} & -p \\
\mathbf{v}_1' & 0 & \ldots & 0 & 0 \\
\mathbf{v}_{j-1}' & 0 & \ldots & 0 & 0 \\
-p' & 0 & \ldots & 0 & 0
\end{bmatrix}
\begin{bmatrix}
dx \\
d\lambda_1 \\
d\lambda_{j-1} \\
d\lambda_0
\end{bmatrix}
= \begin{bmatrix}
\lambda_0 \\
0 \\
0 \\
E
\end{bmatrix}
\]

(A1)

where \( W = (w_{rs}) \), \( w_{rs} = \partial U_r / \partial x_s \), \( \mathbf{v}_i = (u_{i1}, \ldots, u_{in})' \), and \( E = -dY + \sum_s d\lambda_s \). Noting that \( w_{rs} = w_{sr} \), the coefficient matrix is symmetric. Let \( D \) be the determinant of this matrix and \( D_{rs} \) the cofactor of its \((r, s)\) element. Then, analogous to standard results, one has

\[
\frac{\partial x_s}{\partial p_r} = (\lambda_0 D_{rs} + x_r D_{n+j,s}) / D
\]

(A2)

\[
\frac{\partial x_s}{\partial B} = -D_{n+j,s} / D.
\]

(A3)

Suppose \( dp_r \neq 0 \) for a particular \( r \) and \( dp_s = 0 \) for all \( s \neq r \), and choose \( dY \) so that \( du_j = \sum_s u_{js} dx_s = 0 \). Then from (12) and (14),

\[
\lambda_0 \sum_s p_s dx_s = \sum_s \sum_i \lambda_i u_{is} dx_s
\]

(A4)

after putting \( du_j = 0 \). But from each equation in (6), \( \sum_s u_{is} dx_s = 0 \) and therefore both sides of (A4) vanish. Hence \( E = 0 \) in (A1) so

\[
\frac{\partial x_s}{\partial p_r} \bigg|_{V=\text{const}} = \frac{\lambda_0 D_{rs}}{D}.
\]

(A5)
Finally, substituting (A5) and (A3) in (A2) gives (16).
Appendix B

Suppose \( x_a = 0 \) to begin with. Notice that the very first unit of \( a \) substituted for \( b \) units of \( \beta \) means a nonbinding constraint in (6) if \( u_{ia}/u_{i\beta} > b \), making the corresponding \( \lambda_i = 0 \). Writing \( c_a = \sum_{i=1}^{j-1} \lambda_i u_{ia} \) and \( c = c_a/c_{i\beta} \), we therefore have \( c = b \). From (14),

\[
\begin{align*}
&u_{ja} + c_a - p_a \lambda_0 = 0 \\
&cu_{j\beta} + c_a - cp_{i\beta} = 0
\end{align*}
\]

hence

\[
(u_{ja} - cu_{j\beta}) = (p_a - cp_{i\beta}) \lambda_0.
\]

Since \( b = c \), and \( c < p_a/p_{i\beta} < u_{ja}/u_{j\beta} \) from (14) and (21),

\[
(u_{ja} - bu_{j\beta})/(p_a - bp_{i\beta}) = \lambda_0.
\]

This shows that if the choice before \( dY > 0 \) contains \( a \) and is optimal, the \( u_j \) value per dollar yielded by the substitution is equal to \( \lambda_0 \), which as usual gives the sensitivity of the solution value of the maximand to the constraint constant \( Y \).

The argument in the text is couched so that it can be used repeatedly in the later discussion, particularly for Proposition 5.
Notes

1. This important paper has been relatively neglected in the literature, but see e.g. Chipman (1960), Encarnación (1964, 1983), Day and Robinson (1973), and the survey article by Fishburn (1974).

2. Cf. Simon (1955) who suggested the interpretation of choice as a vector but did not indicate how vectors would be ordered.

3. Although we take it as given, the \( u_i \) level corresponding to \( u_i \) is explainable in a more complete framework by biological and individual factors or by socially determined norms. One speaks of enough food, a dish with enough seasoning, a large enough variety of dishes for guests at a feast, etc.

4. The idea is that the real outlay on \( C \) is larger at a higher real income, using the new prices for measuring changes in real terms.

5. The displacements may be less than total, so it is possible for the choice to include three or more \( C \) goods of different qualities, e.g. an ordinary wine for the usual supper, a better one for Sunday dinner, and an even better one for special occasions.

References


