Institute of Economic Development and Research
SCHOOL OF ECONOMICS
University of the Philippines

Discussion Paper No. 66-6 September 24, 1966

ON GROUP DECISIONS INVOLVING RISK

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1928

ON GROUP DECISIONS INVOLVING RISK[1/]

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There is an important class of group decision situations where it seems appropriate, because of common values held in the group, to make stronger assumptions than usual regarding the decision-making process. For example, the board of directors of a business firm presumably evaluates the firm's policies in terms of profit rate, sales volume, public relations, and other variables that reflect the firm's objectives. The economic planning commission in an underdeveloped country judges a development program by such criteria as employment, national income, regional balance, and other widely accepted indices of welfare. Despite common interests and agreement on the objectives, however, in general we may expect different views as to the extent to which the various objectives should be pursued.

In this paper we formulate a hypothesis of group decision-making in risk situations. Following the usual terminology, risk

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1/ The author is grateful for the opportunity made possible by a Rockefeller Foundation grant to spend the academic year 1965-66 in Stanford University. Thanks are also due to Professor Kennet J. Arrow for his comments on an earlier draft; of course the author is solely responsible for the paper.

is involved if specific probabilities are associated with the possible states of nature. We shall assume that in a sense to be made precise, the individuals share certain values and criteria of choice regarding the particular problem on hand. Within this restriction, they may have quite different preferences. The primary aim is to suggest a hypothesis that may have explanatory value,[2] though the model to be presented seems to have also some normative appeal.

I

It is convenient to begin with the certainty case.[3] Let u_i (i=1,2,...) be an "ordinal" index of preference with respect

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2/ The model of group decision presented by Savage some years ago was intended for normative purposes. (See L.J. Savage, The Foundations of Statistics (New York 1954), pp. 172-74.) It may also be mentioned that Savage assumed that the members of the group have the same preference ordering over the possible outcomes, but he laid emphasis on different probability judgments regarding the possible states of nature. Outside of Savage's there seems to be no other attempt in the literature to consider group decisions in the absence of certainty.

3/ See J. Encarnación, "Optimum Saving and the Social Choice Function," Oxford Economic Papers, N.S. Vol. XVI (July 1964), pp. 213-20, on which this section is based.

to the group's ith value. On the assumption of common values in the group, given any particular u_i and any two alternatives x and y, the members agree on which of the following holds: $u_i(x) > u_i(y)$, $u_i(x) < u_i(y)$, $u_i(x) = u_i(y)$. In other words, on the basis of any given criterion there is a unique ranking of the alternatives. Of course the rankings by the various criteria are generally different. The important point is that the outcome of x can be represented by a vector $[u_1(x), u_2(x), \ldots]$ which is the same for every individual.

We may suppose that corresponding to u_i there is a value u_i^{k*} such that if $u_i(x) \geq u_i^{k*}$, k would consider x satisfactory with regard to this criterion. Writing $v^k(x) \equiv [v_1^k(x), v_2^k(x), \ldots]$ where $v_i^k(x) \equiv \min[u_i(x), u_i^{k*}]$, our assumption here is that k's preference ordering of the x's is given by the lexicographic ordering of the $v^k(x)$'s. That is, k prefers x to y if, and only if, the first nonzero difference $v_i^k(x) - v_i^k(y)$, $i = 1,2,\ldots$, is positive. Accordingly, k's aim would be to attain as many of his u_i^{k*}'s as possible, starting with the most important; he would therefore maximize some $u_j(x)$ subject to the appropriate constraints on $u_i(x)$ for $i < j$. The individual preference orderings are generally different, depending on the u_i^{k*}'s.

The group preference ordering is most naturally defined in the same way as the individual's, omitting the index k throughout. All that is required is to determine the u_i^*'s, which may be called the group's objectives.

Consider the selection of u_1^* on the basis of individual choices for this parameter. Each individual will want u_1^* to be as close as possible to his own u_1^{k*}, since any greater value would imply an unnecessarily high constraint on u_1 while any lower value would be less than satisfactory. Assuming an odd number of individuals in the group (or an extra tie-breaking vote by the chairman in the case of an even number), the conditions of Professor Black's theorem on single-peaked preferences[4/] are obviously satisfied; hence the median of the u_1^{k*}'s is the only choice for u_1^* that can win by a simple majority against any other possible choice. It is therefore reasonable to assume that the group's u_1^* is the median of the u_1^{k*}'s, or $u_1^* = \text{med}(u_1^{k*})$ for short. A similar argument gives $u_2^* = \text{med}(u_2^{k*})$, etc., and the group decision is defined.

4/ D. Black, The Theory of Committees and Elections (Cambridge 1958), p. 16.

II

In situations involving risk, the outcome of x when s is the true state of nature may be represented by $[u_1(x,s), u_2(x,s),\ldots]$. Given the prior determination of the u_i^*'s, we can consider the additional complications due to the presence of risk.

It was argued in a previous note[5/] that the probability $p_i(x) \equiv Pr[u_i(x,s) \geqq u_i^*]$ is a criterion of choice, and that the parameter p_i^* denotes the probability considered satisfactory for the attainment of u_i^*. The greater is p_i^*, the less is one willing to risk this objective. Although the original context was the decision process of an individual, the same criterion seems reasonable in group decision-making. To illustrate, there is the familiar statistical practice of taking some probability (say 95%, the p_i^*) of detecting batches of goods with more than a certain fraction of defectives (say 2%, corresponding to the u_i^*) as good enough. It was also suggested that $\bar{u}_i(x) \equiv \min_s u_i(x,s)$ is another criterion that becomes relevant in the choice among alternatives when $p_i(x) \geqq p_i^*$. By considering $\bar{u}_i$, one puts a floor under the worst possible results of a decision.

5/ J. Encarnación, "On Decisions under Uncertainty," Economic Journal, Vol. LXXV (June 1965), pp. 442-44.

Writing $V(x) \equiv [V_1(x), V_2(x), \ldots]$, where

$$V_{2i-1}(x) \equiv \min[p_i(x), p_i^*]$$

$$V_{2i}(x) \equiv \min[\bar{u}_i(x), u_i^*] \qquad (i=1,2,\ldots)$$

we use the hypothesis in the paper cited and assume that the lexicographic ordering of the $V(x)$'s defines the group preference ordering. The individual preference orderings are of course to be defined in a similar way; specifically, the components in individual k's choice vector are

$$V_{2i-1}^k(x) \equiv \min[p_i^k(x), p_i^{k*}]$$

$$V_{2i}^k(x) \equiv \min[\bar{u}_i(x), u_i^{k*}]$$

where $p_i^k(x) \equiv Pr[u_i(x,s) \geq u_i^{k*}]$. We need to consider how the group parameters p_i^* may be determined.

Having fixed u_1^* earlier, consider the possible choices for p_1^*. In general, k's choice for p_1^* will be different from his p_1^{k*} since u_1^* is now the relevant parameter. If $u_1^{k*} < u_1^*$, k may be expected to express a choice for p_1^* -- let us call it p_1^{*k} -- that is less than p_1^{k*}, for k finds u_1^* unnecessarily high and so its attainment can be exposed to greater risk. If $u_1^{k*} > u_1^*$, k would want a higher probability that the group decision should result in u_1^* or better, in which case $p_1^{*k} > p_1^{k*}$.

We say, therefore, that in a group decision context, k considers x satisfactory with respect to p_1 if $p_1(x) \geq p_1^{*k}$. Since the individual preference rankings of possible choices for p_1^* are single-peaked, Black's theorem is applicable again and we put $p_1^* = \text{med}(p_1^{*k})$. Defining the other p_i^*'s in a similar way completes the determination of the group preference ordering.

Although all this may seem excessively involved, it is not necessary to suppose that the members of the group have first to establish the p_i^*'s before a group decision could be reached. A relatively simple procedure gives the group's choice between any two alternatives, as indicated by the following implication of the assumptions:

The group prefers x to y if, and only if, in the sequence of criteria $(c_1, c_2, c_3, c_4, \ldots) \equiv (p_1, \bar{u}_1, p_2, \bar{u}_2, \ldots)$, there is a c_j such that $c_j(x) > c_j(y)$ and a majority finds y less than satisfactory with respect to c_j, and for every $h < j$, $c_h(x) = c_h(y)$ or else a majority considers both x and y satisfactory on the basis of c_h.

To see this, note that the underscored statement implies, for $h = 2i-1$, that $p_i(x) \geq p_i^*$ and $p_i(y) \geq p_i^*$ since $p_i^* = \text{med}(p_i^{*k})$; for $h = 2i$, $\bar{u}_i(x) \geq u_i^*$ and $\bar{u}_i(y) \geq u_i^*$ since $u_i^* = \text{med}(u_i^{k*})$. The "if" part of the proposition therefore means that $V_j(x) > V_j(y)$

and $V_h(x) = V_h(y)$ for all $h < j$, which defines the group preference.

The importance of the implication lies in the operational significance of the hypothesis. In effect it says that the group arrives at a choice between alternatives even without any explicit knowledge of the p_i^*'s. If a majority of the group considers both x and y satisfactory with respect to the probability of attaining the first objective, they are compared on the basis of their worst possible outcomes. Should the alternatives be equal in this regard, the probability of reaching the second objective becomes the criterion of choice. Again, if a majority (which may be different from the preceding majority) finds x and y satisfactory, the next criterion is considered, etc., until there is a majority vote to eliminate one alternative as being inferior to the other.

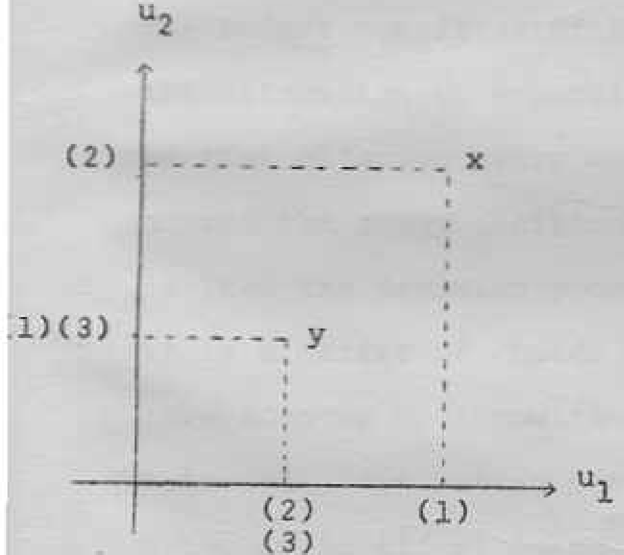
The above procedure seems quite consistent with what we know of actual group decision processes. Groups do in fact consider issues one at a time, and issues are settled usually by majority vote. What is often oversimplified in the analysis of choice is the fact that an alternative is really a multidimensional affair. Under our hypothesis, the group's decision is reached by considering the various dimensions of choice in their order of importance. The decision can be viewed, therefore, as built up from a series of separate ones that are made, interestingly enough, by simple majority rule.

III

It should be emphasized that the group's decision is a function of the individual choice parameters -- it is not a function of individual choices among the alternatives as such. This raises the possibility (at first sight awkward) that in some situations, everyone in the group may prefer x to y and yet group preference is for y over x. For instance, consider three individuals with choice parameters as indicated in the diagram. On each u_i-axis, (k) denotes the value u_i^{k*}. Individual 1 prefers x to y, or xP^1y, since $u_1(y) < u_1(x) = u_1^{1*}$. Individual 2 finds both x and y satisfactory by u_1; looking at u_2, xP^2y. Finally, xP^3y. But the group chooses y over x because both alternatives meet the first three objectives and $u_4(x) < u_4(y) = u_4^*$.

Such a violation of the "Pareto principle"[6] -- that if everyone prefers x to y, so does the group -- would seem to be a relatively infrequent phenomenon in actual decision-making.

6/ K.J. Arrow, Social Choice and Individual Values, 2nd ed. (New York 1963), p. 96.


u_2
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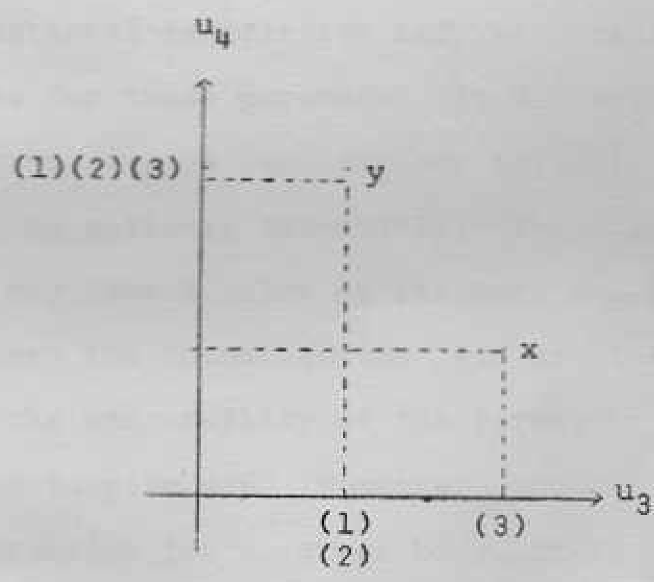
u_4
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when it does occur, however, an explanation would be called for, and this seems provided by our hypothesis.

From a normative viewpoint, the Pareto principle has great intuitive force. Nonetheless it could be argued that occasional violations of this principle may be tolerated and even desired by the members of a group. In the illustration above, suppose that u_1 refers to national defense, u_2 to health and welfare services, u_3 to public works, u_4 to education, and the budget committee of the national legislative body is deciding the allocation of expenditures for these purposes. It may well be that although every member prefers x to y, they may all accept the group decision y as entirely reasonable. The point is that the decision process may have a value of its own, and if it is a matter of choice between the "constitution" (to use the term adopted by Arrow[7/]) and the universality of the Pareto principle, the latter may have to give way. Moreover, when a group is in effect making a decision for a larger body, it is quite possible for each member of the group to want a group decision different from his own, if there is a likelihood that the larger body would make the same decision as the group's.

7/ Ibid., p. 105

IV

To conclude, we note that as one should require, the formulation in Section I is the special case of Section II's where one state of nature has probability 1 and every other state has probability 0. In this case, $p_i(x)$ is either 0 or 1. If $p_i(x) = 0$, $V_{2i}(x) = \bar{u}_i(x) = u_i(x)$; if $p_i(x) = 1$, $V_{2i}(x) = u_i^*$. The situation for the individual is of course similar.