INFLATION, GROWTH AND POLICY COORDINATION

Renato E. Reside, Jr.\textsuperscript{b} and Parantap Basu\textsuperscript{c}

The effect of monetary policy on output growth and inflation is examined in a monetary endogenous growth model with an explicit banking sector that intermediates capital. Monetary policy is coordinated with fiscal policy in the sense that all the seigniorage revenue gleaned through the inflation tax is spent to make efficient provision of public services. With the reserve requirement and money growth as instruments, policy coordination implies that one instrument may be solved endogenously when the other is predetermined. A higher reserve requirement being a binding constraint on capital creation generates adverse growth effects. On the other hand, by endogenously reducing the money growth rate, a higher reserve requirement lowers inflation. Interestingly, when public services are not productive, policies need not be coordinated. In this scenario, we derive a non-standard result wherein a rise in reserve requirements raises inflation. We conclude with the suggestion that differences in the degree of policy coordination account for cross-country differences in the persistence of inflation.

1. Introduction

There are now two distinct branches of macroeconomics literature dealing with policy-induced effects on growth and inflation. One strand of literature focuses on the key role played by fiscal policy in determining growth (see for example, Jones and Manuelli (1990), Barro (1990) and Rebelo (1991)). The other strand of the macroeconomics literature emphasizes the effect of monetary policy on growth. As far as this second line of research is concerned, the

\textsuperscript{a} This paper is an extension of Chapter 3 of Reside's (1996) dissertation thesis. Authors would like to acknowledge Bruce Champ for valuable suggestions. The usual disclaimer applies.

\textsuperscript{b} Department of Economics, University of the Philippines School of Economics, Diliman, Quezon City, Philippines. E-mail: renato_reside@hotmail.com.

\textsuperscript{c} Department of Economics, Fordham University, 311 E. Fordham Road, Bronx, NY 10458, USA. Tel. (718) 817-4061. E-mail: Basu@Murray.Fordham.Edu.
real business cycle school highlights the inflation tax consequences of a change in monetary aggregates (see, for example Cooley and Hansen (1989)). Roubini and Sala-i-Martin (1992, 1995) focus on the adverse effects of various kinds of bank regulations including reserve requirements on growth and find that these regulations, by impairing the efficient functioning of the financial intermediaries, adversely affect growth. Chari, Jones and Manuelli (1995) explicitly model the banking sector and reach a very similar conclusion.

Although there is considerable theoretical work to analyze the effect of fiscal and monetary policy on growth, very few papers attempt to analyze the growth and inflationary effects of monetary policy when monetary policy is coordinated with fiscal policy as far as financing fiscal spending is concerned. Most of the monetary growth models tend to assume that monetary injections appear in the household’s budget constraint in the form of a lump-sum transfer which has no allocative effects on the economy. On the other hand, in the branch of literature dealing with fiscal policy, government spending is assumed to be financed either by income taxes (Barro, 1991) or bond financing (Kasa, 1994) or both (Dotsey and Mao, 1994). This artificial separation between fiscal and monetary policy is still extant in the macroeconomics literature although the literature has advanced enormously with the advent of dynamic general equilibrium modeling.

The major goal of this paper is to bridge this gap in the literature by paying special attention to the destination of the seigniorage revenue and the way in which it is used. In this paper, we examine two scenarios, one in which seigniorage revenue is used for productive purposes and another case where it is not. In the former, the government uses seigniorage revenue to provide public services to aid private production. In the latter, public services are unproductive and seigniorage is merely wasted.

Barro (1990) has demonstrated the role of government in the growth process. While public services in Barro’s model are financed by income tax, in our model, it is financed by seigniorage revenue. This complication is important because we are interested in knowing the effect of monetary policy on growth when there are some elements of coordination between monetary and fiscal policy. Here, the policy coordination is modeled by assuming that all inflation
tax revenues are used to make efficient provision of public services. The level of public service is determined in our model by invoking the natural efficiency condition which involves marginal cost pricing. On the other hand, when public services are unproductive, the level of public service need not be determined by any efficiency condition.

We consider a monetary endogenous growth model incorporating congestion effects of public services a la Barro and Sala-i-Martin (1992). Like Chari et al. (1995), our model also contains an explicit banking sector which fully intermediates the capital stock in the economy. Just as in any policy coordination model, one of the central bank's policy instruments has to be endogenous to achieve the desired policy coordination objective. In the policy coordination model, the central bank has two policy instruments, the reserve ratio and the growth rate of money stock. In the policy coordination case, we consider two scenarios. One, where the monetary authority first sets the reserve requirement and lets the money growth rate adjust endogenously to satisfy the government budget constraint ensuring that the entire seigniorage revenue is spent to make efficient provision of public service. In the second scenario, the money growth rate is predetermined and the reserve ratio endogenously changes in response to any exogenous change in the money growth rate. In the non-coordination case (when public services are unproductive), there is no need to invoke the efficiency condition in order to solve for the optimal provision of public services. Thus, both the reserve ratio and the money growth rate become free parameters. We conclude by suggesting that the coordination case can account for developed country behavior, while the non-coordination case accounts for developing country behavior.

The monetary transmission mechanism in the policy coordination case is very different from the non-coordination case (and from the transmission mechanisms in other papers, such as Roubini and Sala-i-Martin (1992) and Chari et al. (1995)). In the coordination case, exogenous movements in either the reserve requirement or the money growth rate artificially alter the monetary base, the base for the inflation tax or seigniorage. Therefore, in order to satisfy the government budget constraint (which requires that seigniorage revenue be equal to the efficient level of public service),
the central bank has to alter the other instrument, which is endogenously determined. For example, an exogenous rise in the reserve requirement artificially boosts the monetary base, which raises seigniorage unless there is an offsetting endogenous fall in the money growth rate to lower inflation. Although lower money growth partly alleviates the inflation tax burden on the household, it is not enough to compensate for the adverse real effect of a higher reserve ratio. The reserve requirement acts as a binding constraint on banks in the process of intermediating capital and as a result, steady state growth in the economy declines.

In the non-coordination case, the government is not constrained to offset exogenous changes in either instrument. Thus, an exogenous rise in reserve requirements without an offsetting endogenous fall in money growth, only serves to lower the growth rate of output, which raises inflation for any given money growth rate.

Since movements in output are determined by the movements of the reserve requirement (which determine the volume of credit for the purchase of capital), the monetary transmission process closely resembles the bank lending channel of the credit view of the transmission mechanism described in Bernanke and Gertler (1995). The difference between the mechanism here and those in other papers is the way in which inflation is affected.

The rest of the paper is organized as follows. In section 2, we lay out our policy coordination model with self-sustained growth. Section 3 reports simulation results to determine the qualitative properties of the coordination model. Section 4 discusses how non-coordination of policies can arise and their implication for inflation and growth. Section 5 is a synthesis of the implications of both models. Section 6 ends with concluding comments.

2. A Model of Policy Coordination

We assume that the economy is populated by N households, each of which owns a firm. The production function is of the following “AK” form with a congestion externality (Barro and Sala-i-Martin, 1992) which permits endogenous growth:
INFLATION, GROWTH AND POLICY COORDINATION

(1) \[ Y_{it} = AK_{it} \left( \frac{G_t}{Y_t} \right)^\theta \]
where \[ \sum_{1}^{N} Y_{it} = y_{it}, N \text{ is fixed} \quad i = 1, \ldots, N \]
where \( Y_{it} \) = output of the \( i \)th firm and \( K_{it} \) is a broad measure of capital, including physical and human capital and perhaps even stock of knowledge used by the \( i \)th firm. \( A \) is total factor productivity, and for a given quantity of aggregate public services, \( G_{t} \), the quantity available to an individual declines as other users congest the facilities. The productivity of public services is reflected in \( \theta \) (which is assumed to be positive).

Aggregate public services, \( G_{t} \) (which includes the provision of infrastructure, police and fire services, etc.), can alternatively be viewed as an intermediate input to the private sector production process as in Barro (1990).\(^1\) We assume that the quantity of \( G_{t} \) available to firms declines as other users congest facilities (or, as \( Y_{t} \), aggregate income rises). Also, agents view aggregate \( G_{t} \) as given when formulating optimizing decisions. Thus, in the per capita production function, agents use aggregate public services instead of per capita public services.

Since each household owns a firm, we can convert (1) into per capita form:

(2) \[ y_{t} = A k_{t} \left( \frac{G_t}{Y_t} \right)^\theta \]

where \( y_{t} \) and \( k_{t} \) are per capita versions of their upper case counterparts. Letting \( A_{0} = (A/N^\theta) \), we can convert this into

---

\(^1\) Barro's case for including \( G \) as a separate argument of the production function stems from the notion that private inputs (capital and labor) are not a close substitute for public inputs. This is especially the case when user charges are difficult to implement, such as the case of national defense and the maintenance of law and order. Thus, in this model, we assume that public services are not subject to user charges. We can think of the government purchasing a portion of private output and then using these purchases to provide free public services to private producers.
RENATO E. RESIDE, JR. AND PARANTAP BASU

\[(3) \quad y_t = (A_0 k_t G_t^\theta)^{1+\theta} = A_0^{1+\theta} k_t^{1+\theta} G_t^{1+\theta}.\]

The per capita production function thus exhibits diminishing returns to the private input, broad capital, but constant returns to scale to the public and private inputs together \((k_t \text{ and } G_t)\) for fixed \(L\) (since the powers to which \(k_t\) and \(G_t\) are raised add up to one).

If factor markets are competitive, the real interest rate in this economy equals the marginal product of capital:

\[(4) \quad \frac{\delta y_t}{\delta k_t} = r_t = MPK_t = \left( \frac{1}{1+\theta} \right) A_0^{\frac{1}{1+\theta}} \left( \frac{G_t}{K_t} \right)^{\theta \frac{1}{1+\theta}}.\]

Since public services appear as an intermediate input in the production function, value-added by definition is:

\[(5) \quad (A_0 k_t G_t^\theta)^{1+\theta} - G_t.\]

The efficient provision of public services requires \(G_t\) to be set at a level which maximizes value-added. In other words, if the government wishes to levy an optimal tax, it must follow a marginal cost pricing rule which requires that the marginal product of public services \((G_t)\) equal unity. This implies that:

\[(6) \quad G_t = \left( \frac{\theta}{1+\theta} \right) y_t\]

of alternatively using (3),

\[(7) \quad G_t = \left( \frac{\theta}{1+\theta} \right)^{1+\theta} A_0 k_t\]

where
INFLATION, GROWTH AND POLICY COORDINATION

(8) \[ y_t = A_0 k_t \left( \frac{\theta}{1+\theta} \right) \]

The real interest rate is, therefore,

(9) \[ r_t = A_0 \left( \frac{\theta}{1+\theta} \right). \]

Note that the efficient level of \( G_t \) in (6) must be proportional to the private sector output, \( y_t \). The government may resort to alternative fiscal or monetary policy tools to finance this public service. If the government resorts to income taxation a la Barro (1988), the optimal income tax rate in this model will be \( \theta / (1+\theta) \). In our model, however, we assume that this public service is financed by inflation tax to enable an effective coordination between fiscal and monetary policy. This is an issue upon which we will elaborate in section (2.4) in relation to the government budget constraint.

Regardless of what tax instrument the government uses, the efficient provision of public service requires that the government must acquire \( \theta / (1+\theta) \) fraction of the aggregate output. It follows, therefore, that ex post after-tax real income is \( y_t - \left( \theta / (1+\theta) \right) y_t \). This is equivalent to the value-added expression in (5). Using (7) and (8), the ex post after-tax real income is

(10) \[ \bar{y}_t = A_0 k_t \left( \frac{\theta}{1+\theta} \right) \left( \frac{1}{1+\theta} \right) \]

and therefore, the ex post after-tax real interest rate is

(11) \[ \bar{r}_t = A_0 \left( \frac{\theta}{1+\theta} \right) \left( \frac{1}{1+\theta} \right). \]
2.1 Households

Households are assumed to be families of the Sidrauski-type (i.e., have money in the utility function). Households derive utility from holding real money balances. Households are assumed to have Cobb-Douglas preferences between consumption and real money balances according to the following constant relative risk aversion utility function

\[(12) \quad U_t = \int_0^\infty e^{-pt} \frac{(c_t^\delta m_t^{1-\delta})^{1-\lambda} - 1}{1 - \lambda} \, dt\]

where

\[U_c, U_m > 0, \quad U_{cc}, U_{mm} < 0\]

where \(c_t\) is per capita consumption and \(m_t\) is per capita real money balances. \(\delta\) is the share of consumption in utility, \(p\) is the subjective rate of time preference (the discount rate) and \(\lambda\) is the (constant) relative risk aversion parameter and the elasticity of marginal utility.

In this economy, there is no uncertainty. Also, individual wealth consists of nominal money holdings, \(M_t\), and nominal holdings of bank deposits, \(D_t\), from which they derive interest income. Individuals own firms and therefore produce all of the real output in the economy, from which they derive another part of their income. In order to procure capital, \(K_t\), individuals borrow from banks in the amount \(B_t\) which they retire next period with interest. However, banks are also assumed to be owned by the households and, therefore, individuals also have claims to bank profit, \(\Pi_t\).

Households thus maximize the functional (12) subject to the following aggregate budget constraint:

\[(13) \quad \frac{P_t C_t + (1 + R_{bt}) B_t + M_{t+1} + D_{t+1} = P_t AK_t \left( \frac{G_t}{Y_t} \right)^{\theta}}{+ M_t + (1 + R_{dt}) D_t + B_{t+1} + \Pi_t}\]
where $C_t$ is real consumption (hence, it is multiplied by $P_t$, the price level, to yield nominal consumption), respectively. Bank interest rates are determined in the following Fisherian manner: $R_{bl}$, the nominal interest rate on bank loans, is equal to $r_{bl} + \pi_t$, the real lending rate $r_{bl}$ plus the rate of inflation, $\pi_t$. $R_{dt}$, the nominal interest rate on deposit holdings, is equal to $r_{dt} + \pi_t$, the real deposit rate $r_{dt}$ plus the rate of inflation. The derivation of interest rates will be discussed in the next section.

The left hand side of (13) represents uses of current period wealth. This includes consumption, payment of interest on current period loans, and holdings of next period money balances and time deposits. The right hand side of (13) refers to sources of current period wealth, which include revenues from the sales of current output, current period money balances, current period bank time deposits plus interest, new loans, and bank profits.

2.2. The Banking Sector

Banks are assumed to have two assets in their balance sheets: loans, $B_t$, and required reserves. Required reserves mandated by the Central Bank are some fraction $\nu$, of total deposits, $D_t$, to be deposited with the Central Bank. Furthermore, banks are assumed to have a single liability: deposits. For simplicity, if we ignore bank capital, then the bank balance sheet identity is

\[ (I - \nu)D_t = B_t. \]

Banks cannot loan all of their deposits out because they have to keep $\nu D_t$ of them in reserve as deposit with the central bank.

Nominal bank profit is given by:

\[ \Pi_t = R_{bl} B_t + R_{vdt} \nu D_t - R_{dt} D_t \]

Bank profit, $\Pi$, equals nominal interest earned on loans $R_{bl}B_t$, plus nominal interest earned on holding reserves $R_{vdt}\nu D_t$, minus nominal interest paid out to depositors, $R_{dt}D_t$. Now, as is the usual case, the Central Bank sets $R_{vdt}$ equal to zero, so that nominal bank
RENATO E. RESIDE, JR. AND PARANTAP BASU

profit reduces to the first and last terms on the right hand side of (15). Also, since \( R_{vd}t = 0 \), then it follows that the real return to holding reserves equals \(-\pi_t\). Thus, for commercial banks (and their owners), holding reserves is costly, but to the government, reserves represent an addition to the base for the inflation tax revenue besides real money balances.

In a competitive environment, bank profits in (15) are set equal to zero which implies that

\[
(16) \quad \frac{R_{dt}}{1 - \nu} = R_{bt}.
\]

The reserve requirements drive a wedge between the nominal lending and nominal deposit rates for banks. That is, a higher reserve requirement raises the nominal lending rate for any given nominal deposit rate. The reason here is that when the deposit rate is given, intermediaries have to be compensated for holding reserves (which are non-earning assets) in terms of being able to charge higher nominal lending rates.

We can solve for the equilibrium relationship between real lending and deposit interest rates in this model by using (16):

\[
(17) \quad r_{bt} = \left( \frac{1}{1 - \nu} \right) [r_{dt} + \pi_t \nu]
\]

Higher reserve requirements and higher inflation raise the real lending rate for any given \( r_{dt} \). Later, we will show that the rate of inflation is a positive function of the reserve requirement, so that changing reserve requirements has both direct and indirect effects on the real lending rate.

2.3 The Government

The government budget constraint ensures that all the seigniorage revenues from levying the inflation tax on the monetary base \( z \) which will be precisely characterized later on, are
spent to make for the efficient provision of public services. This means that

(18) \( G = \pi z \).

Since optimal \( G \) equals \((\theta/(1+\theta))y\) by (6), (18) can be rewritten as

(19) \( \pi z = \left( \frac{\theta}{I + \theta} \right) y \).

If \( \mu \) equals the central bank-controlled constant growth rate of nominal money balances \( \frac{\dot{M}}{M} \), then by definition,

(20) \( \pi = \mu - \frac{\dot{m}}{m} \) where \( \mu = \frac{\dot{M}}{M} \)

which means (19) reduces to:

(19a) \( (\mu - \sigma)z = \left( \frac{\theta}{I + \theta} \right) y \)

where \( \sigma \) is the steady state growth rate of \( m \) which we will derive in the next section.

Before analyzing the steady state equilibrium of this model, it is important to understand how we close the model. Since the household does not internalize \( G \) in its decision problem, the household solves its optimal consumption and savings decision treating \( \mu \) and \( \nu \) as parametrically given. This optimal consumption-saving decision generates a steady state growth rate \( \sigma \). Thus, in principle, \( \sigma \) is a function of \( \mu \) and \( \nu \). Given such a steady state value of \( \sigma \), there are numerous combinations of \( \mu \) and \( \nu \) which satisfy the government budget constraint (19a).
Two possible scenarios are of interest here. One where the reserve ratio \( v \) is predetermined and \( \mu \) adjusts endogenously to satisfy (19a). This relationship between \( \mu \) and \( v \) is highly nonlinear and cannot be solved analytically and we have to resort to a numerical procedure to assess how an exogenous change in the reserve requirement affects the steady state growth rate via an endogenous adjustment in \( \mu \).

The second policy scenario is where the central bank first exogenously sets \( \mu \). Given \( \mu \), there exists a \( v \) which will satisfy the government budget constraint (19a). We can then analyze how a change in money growth \( \mu \) transmits to the real sector via an endogenous change in \( v \). This latter analysis explicitly addresses the credit view of the monetary transmission mechanism. Before we spell out the two scenarios, it is important to analyze the steady state properties of the policy coordination model, which is done next.

2.4 The Steady State Equilibrium

Rearranging (13) and upon substitution of (15), we note that all interest terms cancel out. In continuous time form, the budget constraint then becomes:

\[
(21) \quad PC + \dot{M} + \dot{D} = PA_0 K \left( \frac{G}{Y} \right)^\theta + \dot{B}.
\]

The following three equations follow by definition

\[
(22) \quad \frac{dM}{dt} = \frac{PL}{m + \pi m}, \quad \frac{dD}{dt} = \frac{PL}{d + \pi d},
\]

\[
\frac{dB}{dt} = \frac{b + \pi b}{PL}
\]

where \( m, d \) and \( b \) are per capita real values of their upper case counterparts. Dividing both sides of (21) by \( PL \), and using (22), we derive the real per capita budget constraint faced by individuals.
\[(23) \quad c + m + \pi m + \ddot{d} + \pi d = \left( A_0 k G^0 \right)^{1+\theta} + b + \pi b. \]

In order to close our model, we assume that the real value of per capita bank loans equals our per capita capital stock; i.e., \( B/PL = k \): all physical capital is financed by borrowing from banks. We have already assumed that all banks are privately owned. Hence, it is necessary to incorporate the following per capita real balance sheet constraint based on (14) into the private budget constraint:

\[(24) \quad (1 - \nu)d = b = k. \]

Note that \( \nu \) becomes a binding constraint to the creation of capital stock. Later, this will be the source of the adverse supply-side effect of \( \nu \) on economic growth.

Given that the monetary base \( (z) \) is composed of money balances and reserves, we can use (22), to get

\[(25) \quad z = m + \nu d = m + \left( \frac{\nu}{1 - \nu} \right) k \]

so that \( z \) is equal to the per capita stock of real monetary base: real holdings of currency plus real holdings of bank reserves which, by the bank balance sheet condition, is a function of the capital stock. Equation (25) also implies that the reserve requirement \( \nu \) is as much a tax on capital stock as it is on deposits.

Using (23) and (25), we can thus derive the simplified budget constraint:

\[(26) \quad \ddot{z} = m + \left( \frac{\nu}{1 - \nu} \right) \ddot{k} = \left( A_0 \left[ \left( \frac{1 - \nu}{\nu} \right) (z - m) \right] G^0 \right)^{1+\theta} - \pi z - c. \]

The state variable is \( z \), the monetary base, and the control variables are \( c \) and \( m \). The modified budget constraint states that real wealth is the accumulation by individuals of high-powered money, the source of which is real output less the inflation tax on the monetary
base (equal to the amount of seigniorage revenue collected by the
government). Since individuals own banks, they own reserves as
well as currency. Since reserves are non-interest bearing assets
deposited at the central bank, they too, are subject to the same
inflation tax as currency in circulation. Households treat $G$, $\mu$ and $\nu$
as parametrically given while maximizing (12) subject to (26). In
equilibrium, $G$ equals $(\theta/(1+\theta))y$ as in (6). Therefore, we note that
even if the government determines optimal values for $\mu$, $\nu$ and/or $G$,
private agents do not internalize these optimal values (i.e., agents
do not include them in their private budget constraint) when optimi-
zing their behavior.

The steady state property of the model is summarized by the
following steady state expressions for the relevant endogenous
variables:

\begin{equation}
\begin{align*}
\text{Economic growth} & \quad \sigma_e = \sigma_m = \sigma_h = \sigma_z = \sigma = \\
& = \left( \frac{1 - \nu}{\nu} \right) \bar{r} - \mu - \rho \\
\end{align*}
\end{equation}

\begin{equation}
\text{Inflation} \quad \pi^* = \frac{\mu\lambda - \left( \frac{1 - \nu}{\nu} \right) \bar{r} + \rho}{\lambda - 1}
\end{equation}

\begin{equation}
\text{Borrowing rate} \quad r_{bt}^* = \bar{r}_t = \left( \frac{1}{1 + \theta} \right) A_0 \left( \frac{\theta}{1 + \theta} \right)^\theta
\end{equation}

\begin{equation}
\text{Deposit rate} \quad r_{du}^* = \left( \frac{1}{\lambda - 1} \right) \left[ \lambda(1 - \nu) \left( \frac{A_0}{\theta} \right) \left( \frac{\theta}{1 + \theta} \right)^{1+\theta} - \nu(\mu\lambda + \rho) \right]
\end{equation}

\begin{equation}
\text{Ex post money demand} \quad \frac{m}{k} = \frac{A_0 - \left( \frac{\nu}{1 - \nu} \right) \mu}{\mu + \frac{1}{\Omega}}
\end{equation}
INFLATION, GROWTH AND POLICY COORDINATION

where

\[ (32) \quad \Omega = \frac{1 - \delta}{\delta} \left( \frac{1}{\left( \frac{1 - \nu}{\nu} \right) \bar{r}} \right) \]

and

\[ (33) \quad \text{Asset composition} \quad \frac{m}{d} = \frac{(1 - \nu)A_0 - \nu \mu}{\mu + \frac{l}{\Omega}} \]

where the monetary instruments \( \mu \) and \( \nu \) abide by the following nonlinear restriction:

\[ (34) \quad \pi^* \left( x + \frac{\nu}{1 - \nu} \right) = \left[ \left( \frac{\theta}{1 - \theta} \right)^{1+\theta} A_0 \right] \]

where \( x \) is the expression for \( m/k \) as in (31). The appendix outlines the derivation of these expressions. Note that both the steady state growth rate and inflation depend on the monetary policy instruments, \( \mu \) and \( \nu \). The presence of the term \((1-\nu)/\nu\) in the growth rate expression signifies the standard deposit creation multiplier process. The deposit creation process is triggered if the household deposits a unit of output at a bank. The resulting excess reserve \((1-\nu)\) generates \((1-\nu)/\nu\) of additional deposits once the deposit creation multiplier process works itself out. Since new deposits give rise to additional loans and loans are costlessly converted into new capital, the growth rate is positively related to the deposit multiplier term \((1-\nu)/\nu\) for a given \( \mu \). Also note from (27) that the balanced growth rate implies that money growth is not supernormal. The reason is that both deposits and loans (and therefore also capital stock) are subject to erosion of value due to the inflation tax. Thus, our model is different from conventional monetary growth models.
From a general equilibrium viewpoint, it is important to observe that we cannot vary \( \nu \) while holding \( \mu \) constant in this model. The restriction (34), which comes from the government budget constraint, makes one of the monetary policy instruments endogenous in our model. The fact that either instrument can be (endogenously) solved as a function of the other (predetermined) instrument reflects the role policy coordination plays in our model. If the central bank sets the reserve ratio exogenously, the monetary growth rate, \( \mu \), must adjust in a way to ensure that the seigniorage revenue is spent to make for efficient provision of public goods (see equations (7) and (18)). On the other hand, if the monetary growth rate \( \mu \) is predetermined, then \( \nu \) must be endogenous to be conformable with the government budget constraint. This endogeneity of the monetary policy instruments makes the comparative statics exercise analytically difficult in our model. That is why we resort to numerical simulations next.

3. Computations and Monetary Policy Evaluations

To calibrate the coordination model, the parameter values for the preference and technology are set at \( \delta = 0.5 \), \( \rho = .04 \), \( \lambda = 2.0 \), \( \theta = 0.2 \). The scale parameter \( A_0 \) is calibrated by setting \( \bar{r} \) in (29) equal to 4\% which is consistent with a benchmark modified golden rule (meaning \( \bar{r} = \rho \)).

3.1. Effect of a change in \( \nu \)

In the next step, we consider the first policy scenario where the reserve ratio is exogenous. There is no published estimate of the average reserve ratio and it varies greatly across countries. In the present context, we search for values of \( \nu \) which generate empirically plausible numbers for the growth rate and inflation. It turns out that given other parameter values, the value of \( \nu \) must be in the neighborhood of 20\% to generate a growth rate of around 3\%. Table 1 reports the computation results for a range of \( \nu \) values from 20 to 30\%. For each value of \( \nu \), \( \mu \) is first calculated from (34) using a nonlinear equation solver. In the next step, \( \pi \), \( \sigma \), \( r_d \), \( m/k \) and \( m/d \) are calculated using equations (27) through (33). Note that \( \mu \) unam-
INFLATION, GROWTH AND POLICY COORDINATION

Biguously decreases as \( v \) increases. This is because of the policy coordination mechanism built into the government budget constraint. When \( v \) increases, it boosts ex post demand for real money balances (which is evident from the values of \( m/d \) and \( m/k \)), thus raising seigniorage revenue. In order to provide the efficient level of public services (financed by seigniorage at the rate \((\theta/(1+\theta))\) of income), the central bank has to reduce money growth. Such an increase in reserve requirement leads to an endogenous reduction in money growth, alleviating the inflation tax burden on the asset holders. However, it is still inimical to the growth prospects for the economy because a higher reserve requirement starves the economy of capital. The indirect positive effect of a lower inflation tax on deposits does not outweigh the direct adverse effect of a higher reserve ratio because higher reserve requirements act as a binding constraint on banks to intermediate capital.

<table>
<thead>
<tr>
<th>( v )</th>
<th>( \mu )</th>
<th>( \pi )</th>
<th>( \sigma )</th>
<th>( r_d )</th>
<th>( m/k )</th>
<th>( m/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.2</td>
<td>.084</td>
<td>.049</td>
<td>.036</td>
<td>.022</td>
<td>.159</td>
<td>.127</td>
</tr>
<tr>
<td>.22</td>
<td>.072</td>
<td>.043</td>
<td>.029</td>
<td>.022</td>
<td>.185</td>
<td>.144</td>
</tr>
<tr>
<td>.24</td>
<td>.062</td>
<td>.038</td>
<td>.024</td>
<td>.021</td>
<td>.219</td>
<td>.162</td>
</tr>
<tr>
<td>.26</td>
<td>.054</td>
<td>.034</td>
<td>.02</td>
<td>.021</td>
<td>.246</td>
<td>.182</td>
</tr>
<tr>
<td>.28</td>
<td>.046</td>
<td>.03</td>
<td>.017</td>
<td>.020</td>
<td>.281</td>
<td>.203</td>
</tr>
<tr>
<td>.30</td>
<td>.040</td>
<td>.027</td>
<td>.013</td>
<td>.020</td>
<td>.321</td>
<td>.225</td>
</tr>
</tbody>
</table>

Although the growth effect of a higher reserve ratio is similar to Roubini and Sala-i-Martin (1992) and Chari et al. (1995), the inflationary effect of a change in \( v \) is exactly the reverse. Note that, primarily because of the endogenous reduction in money growth \( \mu \), the inflation rate declines, making money a more attractive asset to hold. This comovement of money growth rate and inflation is
consistent with the stylized facts documented in several papers including McCandless and Weber (1995). It is also consistent with the conventional demand side effect of a restrictive monetary policy.

Lastly, observe that the loan rate, $r_b$, is unaffected by a change in $v$ because it is evident from (29) that $r_b$ is independent of $v$. However, $r_d$, the real deposit rate, falls. Essentially, banks tend to shift the burden of a higher reserve ratio to depositors in order to break even although this effect is less pronounced because of a compensating decline in the inflation rate. The equilibrium asset composition thus changes in favor of money, which means $m/d$ rises. This flight from deposit to money is basically the root cause of a lower growth rate. Since deposits are proportional to leveraged capital stock by (22), the ratio $m/k$ also rises which means the ratio of monetary base ($z$) to the capital stock ($k$) also rises. An increase in the reserve ratio thus boosts the monetary base in the economy making it an easy source of tax revenue.

Summarizing, for exogenous $v$, the monetary transmission mechanism can be described as:

$v$ rises $\rightarrow$ $m/d$ rises $\rightarrow$ $m/k$ rises $\rightarrow$ $\sigma$ falls.

$v$ rises $\rightarrow$ $\mu$ falls $\rightarrow$ $\pi$ falls.

3.2 Effect of a change in $\mu$

Next, we conduct an alternative policy experiment where the monetary growth rate, $\mu$, is an exogenous policy parameter. Since here we are primarily interested in understanding the nature of the monetary transmission mechanism, we do not attempt to calibrate $\mu$. We basically start from some arbitrary initial value of $\mu$ which gives us plausible numbers for the growth rate and inflation and then increase the value of $\mu$ to determine the growth effect of money supply change. All other parameters are set at the same levels as before.

The results are reported in Table 2. Note that, by causing an endogenous fall in $v$, the increase in the money growth rate impacts real output growth primarily through the credit channel. The re-
reserves ratio, \( r \), is of course endogenous in this scenario to ensure the policy coordination mechanism described earlier. \( r \) decreases in response to an increase in \( \mu \) leading to better intermediation of capital and this promotes growth in the economy. The inflation rate concomitantly increases and because money growth is not supernormal, it has important real effects which operate through the credit channel. The exogenous rise in \( \mu \) leads to an endogenous fall in \( r \), which (because \( r \) is a more potent monetary policy instrument than \( \mu \)) generates a positive growth effect that outweighs the inflation tax generated by inflation. It is also noticeable that asset composition changes in favor of deposits when money supply grows (as \( m/d \) falls). This happens primarily because the endogenous decrease in \( r \) enables banks to hold less reserves. Since households own banks, less reserves mean less money holding from the households’ viewpoint. It thus releases funds for deposits. The equilibrium real deposit rate also increases in spite of an increase in \( \pi \) to make the switch from money to deposits a worthwhile undertaking. The loan rate \( r_b \) stays unaffected because it is independent of \( \mu \).

### Table 2 - Exogenous \( \mu \), endogenous \( r \)

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( r )</th>
<th>( \pi )</th>
<th>( \sigma )</th>
<th>( r_d )</th>
<th>( m/k )</th>
<th>( m/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td>.179</td>
<td>.057</td>
<td>.043</td>
<td>.023</td>
<td>.135</td>
<td>.11</td>
</tr>
<tr>
<td>.12</td>
<td>.158</td>
<td>.067</td>
<td>.053</td>
<td>.023</td>
<td>.113</td>
<td>.095</td>
</tr>
<tr>
<td>.14</td>
<td>.141</td>
<td>.077</td>
<td>.063</td>
<td>.024</td>
<td>.096</td>
<td>.083</td>
</tr>
<tr>
<td>.16</td>
<td>.128</td>
<td>.087</td>
<td>.073</td>
<td>.024</td>
<td>.084</td>
<td>.074</td>
</tr>
<tr>
<td>.18</td>
<td>.116</td>
<td>.097</td>
<td>.083</td>
<td>.024</td>
<td>.075</td>
<td>.066</td>
</tr>
<tr>
<td>.20</td>
<td>.107</td>
<td>.107</td>
<td>.093</td>
<td>.024</td>
<td>.068</td>
<td>.06</td>
</tr>
</tbody>
</table>

For exogenous \( \mu \), the transmission mechanism is summarized:

\( \mu \) rises \( \rightarrow \) \( \nu \) falls endogenously \( \rightarrow \) better intermediation of capital, \( k \) \( \rightarrow \) \( \sigma \) rises.
An important implication of the coordination model is the adverse growth effects of anti-inflationary policy. In order to accomplish a target reduction in inflation, we have to forsake growth. For example, calculations reported in both Tables 1 and 2 reveal that a 1% reduction in inflation is achieved at the cost of sacrificing approximately 1% growth.

4. Non-coordination of Policies: Causes and Effects

At this point, it is important to note the crucial difference between our model and Chari et al. (1995). In Chari et al., government purchases do not serve any productive purpose because all the seigniorage revenues are rebated back to the household in a lump-sum manner. This is equivalent to a situation where $\theta$ is close to 0. Without any public sector externality, there is no restriction on the policy instruments $\mu$ and $\nu$, making $\mu$ and $\nu$ both free parameters as in the Chari et al. model. In this case, an increase in the reserve requirement without any offsetting decrease in $\mu$ (since $\mu$ is now given as constant) lowers the real interest rate on deposits and, from (27), the growth rate of real variables (as in the policy coordination scenario). However, now, the increase in reserve requirement clearly raises inflation, which is evident from (28). This effect of a reserve requirement change on inflation runs contrary to the conventional monetary policy effect which tells us that a restrictive monetary policy lowers inflation. Based on the definition of inflation in (20), inflation rises because, now that $\mu$ is given, there is a fall in the rate of growth for real variables (27). Reside (1996) provides the intuition for this result. From (27), a rise in the reserve requirement lowers the growth rate of output. If the price level did not adjust, then this would lead to a situation of excess

---

2 Note that the value of theta close to zero makes the indirect production function in (8) approach an "AK" form because the limit of ($\theta^9$) approaches unity as $\theta$ approaches zero.

3 Once we explicitly model the policy coordination mechanism, paying special attention to the productive destination of seigniorage revenue, a very different conclusion is reached about the inflationary effect of reserve requirement changes.
INFLATION, GROWTH AND POLICY COORDINATION

demand for goods and services. General market clearing only obtains when the price level rises by enough to correct the situation of excess demand. Hence, the inflation rate jumps.\footnote{Note that in the coordination case, an exogenous rise in \( v \) is offset by an endogenous fall in \( \mu \). This eliminates excess demand and does not lead to a jump in inflation.}

The possibility that \( \theta \) might be close to or equal to zero implies that the inflation tax is being wasted on unproductive public services. Since no restrictions are placed on \( \mu \) and \( v \), it follows that there need be no coordination between monetary and fiscal policies.\footnote{Since the inflation tax is being spent on the purchase of unproductive services, then there is no need for the government to ensure that the inflation tax levied equals the optimal level of tax as in (19).} The government is not constrained to offset exogenous movements in one instrument with endogenous changes in the other instrument (to keep seigniorage equal to the efficient level of public services).

Taking derivatives, we can use comparative statics to determine the effects of an exogenous change in the reserve requirement on the following variables:

\[
\frac{\partial (m/d)}{\partial v} = -\mu (A_0 + \mu) - \left( \frac{\delta}{1 - \delta} \right) \mu \lambda \left[ \left( \frac{2(1 - v) + v^2}{v^3} \right) \right] < 0
\]

(35)

\[
\frac{\partial (m/k)}{\partial v} = -\left( \frac{\mu}{1 - v} \right)^2 - A_0 \left( \frac{1}{1 - v} \right)^2 \left( \frac{\delta}{1 - \delta} \right) \left( \frac{1}{\bar{r}} \right) < 0
\]

(36)

\[
\frac{\partial r_d}{\partial v} = \left( \frac{1}{\lambda - 1} \right) \left[ \lambda \left( \frac{A_0}{\theta} \right) \left( \frac{\theta}{1 + \theta} \right)^{1+\theta} + (\mu \lambda + \rho) \right] < 0
\]

(37)

Equation (35) states that when policies are not coordinated, a rise in \( v \) leads to a change in asset composition in favor of deposits. Meanwhile, from (36) and (37), ex post money demand and the real interest rate on deposits, \( r_d \), declines.
Given the increase in the cost of holding both real money balances and deposits, there is a reasonable explanation for the shift into increasing the proportion of deposits in total assets. Given that the inflation tax (equal to $\pi z$) is merely being wasted on unproductive public services, when a rise in $\nu$ raises inflation, competitive equilibrium should ensure that the inflation tax is minimized. Since the $\pi$ component of the inflation tax has risen, the only way to offset this is to reduce the $z$, or monetary base component of the inflation tax. This is achieved when agents substitute deposits for real money balances. In doing so, agents reduce $z$ since for every peso of real money balances exchanged for $d$, only a fraction of deposits, $\nu d$, are converted into the reserve component of $z$.\(^6\)

The effects of exogenous changes in $\mu$ yield conventional results and are summarized along with the effects of exogenous changes in $\nu$ in Table 3. It can be seen that in the non-coordination case, exogenous changes in $\nu$ and $\mu$ have identical effects except for $\pi$.

### Table 3 - The Effects of Non-coordination of Policies

<table>
<thead>
<tr>
<th>Exogenous rise in $\nu$ (fall)</th>
<th>Exogenous rise in $\mu$ (fall)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$ Given; constant</td>
<td>Given; constant</td>
</tr>
<tr>
<td>$\nu$ Falls (Rises)</td>
<td>Falls (Rises)</td>
</tr>
<tr>
<td>$m/d$ Falls (Rises)</td>
<td>Falls (Rises)</td>
</tr>
<tr>
<td>$m/k$ Falls (Rises)</td>
<td>Falls (Rises)</td>
</tr>
<tr>
<td>$r_d$ Falls (Rises)</td>
<td>Falls (Rises)</td>
</tr>
<tr>
<td>$r_b$ Constant; not affected by $\nu$</td>
<td>Constant; not affected by $\mu$</td>
</tr>
<tr>
<td>$\pi$ Rises (Falls)</td>
<td>Falls (Rises)</td>
</tr>
<tr>
<td>$\sigma$ Falls (Rises)</td>
<td>Falls (Rises)</td>
</tr>
</tbody>
</table>

\(^6\) Furthermore, given that the base for the inflation tax is the monetary base, $z$, and that the monetary base is given by $z = m + \nu d$, it follows that when the monetary base is taxed, holders of real money balances are taxed for 100% of the value of their money holdings, while holders of deposits are taxed for only a fraction, $\nu$, of their deposit holdings. Thus, the shift from money to deposits represents a shift to the asset that bears the lighter inflation tax burden.
5. The Implications of Policy- and Non-Policy- Coordination for Correlation Between Money Growth, Inflation and Output Growth

A noteworthy feature of the simulation results in Tables 1 and 2 is the positive correlation between money growth, inflation and real economic growth in the policy coordination case. By contrast, only money growth and inflation are positively correlated in the non-coordination case. Real economic growth is negatively correlated to both money growth and inflation. Our results are contrasted against the results of studies by Chari et al. and McCandless and Weber (1995) in Tables 4, 5 and 6. All three studies find a positive money growth-inflation correlation.\(^7\) Our policy coordination model predicts a positive money growth-real output growth correlation, which is consistent with the behavior of OECD countries in the McCandless and Weber study. The non-coordination model predicts a negative money growth-real output growth correlation, consistent with other countries (primarily less developed countries). However, unlike the result of the McCandless and Weber study, the correlation between inflation and real output growth is not zero. For the coordination model, the predicted inflation-real output correlation is positive (due to the externality provided by seigniorage-financed public services). As noted by Levine and Renelt (1992) and McCandless and Weber (1995), the true relation between inflation and growth is still uncertain. The fact that the transmission mechanism depicted in the policy coordination model is more consistent with developed OECD countries, while the non-coordination scenario closely resembles the case for LDC's deserves closer empirical scrutiny. Perhaps fiscal and monetary policies are more coordinated and public spending may be less wasteful in the OECD countries. Perhaps non-coordination and low productivity of public services can explain the 1980's persistence of high inflation in Latin American countries despite the presence of high reserve requirements.

---

\(^7\) This positive correlation between money growth and output growth resembles the standard Mundell-Tobin effect although the transmission mechanism is very different from the traditional Mundel-Tobin effect.
Table 4 - Correlation Matrix for Our Model

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\pi$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>- - -</td>
<td>- - -</td>
<td>- - -</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$&gt; 0$</td>
<td>- - -</td>
<td>- - -</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$&gt; 0^*$</td>
<td>$&gt; 0^*$</td>
<td>- - -</td>
</tr>
<tr>
<td></td>
<td>$&lt; 0^{**}$</td>
<td>$&lt; 0^{**}$</td>
<td>- - -</td>
</tr>
</tbody>
</table>

* Denotes policy coordination case
** Denotes the non-coordination case

Table 5 - Correlation Matrix for Chari et al. Model

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\pi$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>- - -</td>
<td>- - -</td>
<td>- - -</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$&gt; 0$</td>
<td>- - -</td>
<td>- - -</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
<td>- - -</td>
</tr>
</tbody>
</table>

Table 6 - Sample Correlation Matrix Observed by McCandless and Weber (1995)

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\pi$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>- - -</td>
<td>- - -</td>
<td>- - -</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$&gt; 0$</td>
<td>- - -</td>
<td>- - -</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$&gt; 0$ (OECD),</td>
<td>$0$</td>
<td>- - -</td>
</tr>
<tr>
<td></td>
<td>$&lt; 0$ (others)</td>
<td>- - -</td>
<td>- - -</td>
</tr>
</tbody>
</table>

In order to account for observed data on inflation and growth, our model seems to be consistent with the following hypotheses:

1) On average, developed OECD countries can be characterized by low (but positive) dependence on the inflation tax and slightly positive productivity of public services.
2) On average, developing countries should be characterized by high dependence on the inflation tax and a very low productivity of public services.

6. Concluding Comment

The primary goal of this paper is to try to account for cross-country differences in inflation and monetary transmission mechanisms. In the process, we have examined two aspects of a single model: one with and one without coordination between fiscal and monetary policy.

Policy coordination is modeled by assuming that all the seigniorage revenue is productively spent to make efficient provision of public goods (which appear as an intermediate input in the production process). The primary difference between the policy coordination model and that of Chari et al. is the presence of policy coordination. This is the direct offshoot of public spending being used for private production. The coordination of monetary and fiscal policies results in a necessity for either reserve requirement or money growth to be determined endogenously for optimal public spending to be achieved. It turns out that the non-coordination scenario yields results similar to the Chari et al. model.

Comparisons of our results with existing studies on inflation, money growth and output growth suggest that the coordination framework is consistent with the behavior of developed countries, while the non-coordination framework is consistent with the behavior of developing countries.

Despite the fact that inflation tax revenues are productively spent in the coordination case, a higher reserve ratio adversely affects growth. In the coordination case, the potential impact of the reserve instrument constraint on loans and capital is stronger than the induced offsetting impact of the monetary growth instrument. In the non-coordination case, the adverse reserve effects on growth are not offset. Because reserve requirement effects are stronger in either case, it follows that this paper is consistent with the credit view of the monetary transmission mechanism.
Appendix

(A.1) \[ H = \frac{(c^\delta m^{1-\delta})^{1-\lambda} - 1}{1 - \lambda} e^{-pt} + \gamma \left( A_0 \left[ \frac{(1-v)}{v} \right] (z - m) \right) G^\theta \right)^{\frac{1}{1+\theta}} - \pi z - c \]

The Hamiltonian is:

(A.2) \[ \frac{\partial H}{\partial c} = e^{-pt} \delta c^{\delta-1} m^{1-\delta} (c^\delta m^{1-\delta})^{-\lambda} - \gamma = 0 \]

(A.3) \[ \frac{\partial H}{\partial m} = e^{-pt} (1 - \delta) c^{\delta-1} m^{1-\delta} (c^\delta m^{1-\delta})^{-\lambda} - \]

\[ \gamma \left[ \left( \frac{1}{1+\theta} \right) A_0 \right]^{\frac{1}{1+\theta}} \left( \frac{G}{k} \right)^\frac{\theta}{1+\theta} \left( \frac{1-v}{v} \right) = 0 \]

and the first-order conditions (FOC) are

(A.4) \[ \frac{\partial H}{\partial m} = e^{-pt} (1 - \delta) c^{\delta-1} m^{1-\delta} (c^\delta m^{1-\delta})^{-\lambda} - \gamma \left[ \left( \frac{1-v}{v} \right) r \right] = 0 \]

and if we substitute (7) for the G/k term in (A.3), we get:

(A.5) \[ \frac{\partial H}{\partial z} = \gamma \left[ \left( \frac{1-v}{v} \right) r - \pi \right] = \gamma \left( \frac{A_0}{\theta} \right) \left( \frac{1-v}{\theta} \right) \left( \frac{\theta}{1+\theta} \right)^{\frac{\theta}{1+\theta}} - \pi = -\gamma \]

The Euler equation is

(A.6) \[ \lim_{t \to \infty} \left[ z(t) \exp \left( -\int_0^t \left( \frac{1-v}{v} \right) r \right) dv \right] = 0 \]

where the last condition is the transversality condition.
INFLATION, GROWTH AND POLICY COORDINATION

The growth equation (27) is derived first by taking logs and then derivatives of (A.2) and plugging it into the Euler equation (A.5). The resulting expression is

(A.7) \[ \sigma_c = \sigma_m = \sigma = \frac{\left(\frac{1 - \nu}{\nu}\right)\dot{r} - \pi - \rho}{\lambda} . \]

Next substitute (A.7) into \( \pi = \mu - \sigma_m \), to find equation (28).

By replacing \( \pi \) in (A.7) with \( \pi^* \) in (28), we end up with the closed-form solution for \( \sigma \), equation (27).

To prove that \( y \), income, and \( k \), capital stock, all grow at the same rate, we take logs and then derivatives of the production function (8). Then, to prove that \( z \), the monetary base, and \( k \) grow at the same rate, we take logs and then derivatives of the government budget constraint (7) to get:

(A.8) \[ \frac{\dot{z}}{z} = \frac{\dot{k}}{k} . \]

To prove that \( z \), the monetary base, \( k \), capital stock, and \( y \), real income, all grow at the same constant rate \( \sigma \), we take logs and then derivatives of the monetary base equation (25) to get:

(A.9) \[ \frac{\dot{z}}{z} = \left( \frac{m}{z} \right) \frac{\dot{m}}{m} + \left( \frac{\nu}{1 - \nu} \right) \left( \frac{k}{z} \right) \frac{\dot{k}}{k} \]

so that the growth rate of \( z \) is the weighted average of the growth rates of \( m \) and \( k \). Next, note that if (A.8) is inserted into (A.9), the equation can be modified into

(A.10) \[ \frac{\dot{z}}{z} = \frac{\dot{m}}{m} = \sigma . \]
Since \( m \) grows at the constant steady state rate \( \sigma \), this means all real variables in our model grow at the constant rate \( \sigma \).

The real borrowing rate (29) is an equilibrium condition that results from the fact that loans equals capital in the model. The real deposit rate (30) is derived by inserting (28) and (29) into (15).

The ex post money demand function (31) is derived by inserting the government constraint (7) into the private budget constraint (26):

\[
(A.11) \quad A_0 \frac{1}{I+\theta} \left[ A_0 \left( \frac{\theta}{1+\theta} \right)^{I+\theta} \right]^{\frac{\theta}{I+\theta}} \pi z - c = \dot{z}.
\]

Replacing \( z \) with (25), and replacing \( m \) with (31), we can derive

\[
(A.12) \quad \left[ A_0 - \pi^* \left( \frac{v}{1-v} \right) \right] \kappa - \pi^* \Omega c - c = \dot{m} + \left( \frac{v}{1-v} \right) \dot{k}.
\]

Divide (A.11) by \( c \). Next use (A.2) and (A.3) to get a proportional relationship between \( m \) and \( c \) which after plugging into (A.12) and using the steady state growth equation (A.7) we obtain the asset composition expression, \( m/d \) in (33).

To get (34) in (19a), replace \( z \) with (25), and replace \( \gamma \) with (7), divide \( k \) and use (33) to substitute out \( m/k \).
References


