Does it pay to be “more rational”?

Romy Balanquit
Pancho Labao
UP School of Economics
Bounded rationality

Why economists have long expressed dissatisfaction with the full rationality assumption that is so pervasive in economic theory?

- Most economic agents are not in fact maximizers
- Many maximization procedures are very difficult to carry out in practice
- Experiments show that people often fail to conform to basic conclusions of rational decision theory
- Results of rational analysis sometimes seem unreasonable even on the basis of simple introspection.
Does it pay to be “more rational”?

Not all the time!

Under full-rationality: There is no issue as to who is more rational

Under bounded rationality: It is an unavoidable question
Who is more rational?

Abe  
93% rational

Bob  
65% rational
Who is more rational?

Abe

Bob

- Can more information really lead to higher payoff?
Assumptions:

- No constraint on computability capacity of individuals (i.e. no information overload or infoxication)
- Marginal utility from additional info is positive and constant
- Each information is distinct and has the same quality (i.e. no info is superior than the other)
- Information sets are tight (i.e. there is no superfluous info in the information set of agents)

All other things constant, we want to see only the effect of the size of the information endowment of an individual on her payoff.
Rock-Paper-Scissors Game

Nash Equilibrium: \[ \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right), \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) \]
Rock-paper-scissors-lizard-spock

Each option:
(i) can defeat two other options and
(ii) can be defeated by another two
For $n=15$, each option can defeat 7 and can be defeated by 7 other
Literature:

A. Both are unbounded:

Player 1

- Rock
- Lizard
- Paper
- Spock
- Scissors

Player 2

- Rock
- Lizard
- Paper
- Spock
- Scissors

B. One is unbounded, the other bounded:

Player 1

- Rock
- Lizard
- Paper
- Spock
- Scissors

Player 2

- Rock
- Lizard
- Paper

C. Both are bounded:

Player 1

- Rock
- Spock

Player 2

- Rock
- Lizard
- Paper
Optimization:

\[ z = f(x, y) \]

\[ x = f(x_1, x_2, x_3; y_1, y_2, y_3) \]

\[ x = 2 \]

\[ y = 1 \]

\[ \Pi_x = f(x_1, x_2, x_3; y_1, y_2, y_3) \]

\[ \Pi_x = f(R, P, R; S, S, S) \]

\[ \Pi_x = 2 \quad \Pi_y = 1 \]

\( f \) is qualitative
Genetic Algorithm (GA)

- a method for solving optimization problems based on a natural selection process that mimics biological evolution.

- the algorithm randomly selects individuals from the current population and uses them as “parents” to produce “children” for the next generation.

- over successive generations, the population evolves toward an optimal solution.
Typical optimization problem using GA

- \[ z = f(x, y) = 3*(1-x)^2*\exp(-(x^2) - (y+1)^2) - 10*(x/5 - x^3 - y^5)\exp(-x^2-y^2) - 1/3*\exp(-(x+1)^2 - y^2). \]
• GA process:

Initial population 5th generation 10th generation
Simplifying analogy

- Chromosome \leftrightarrow \text{Strategy (7-bit strategy)}
- Genes \leftrightarrow \{\text{R, P, S, etc.}\}
- Genotype \leftrightarrow (\text{R, P, S, S, P, S, S})
- Phenotype \leftrightarrow \{1,0,-1\}
GA Mechanism

1. Initialization: generate an initial population at random (i.e. $P(i)=P(1)$, first generation).

2. Iterate:
   a) evaluate the fitness of individuals in $P(i)$
   b) select parents from $P(i)$ based on their fitness
   c) generate offspring from the parents using crossover & mutation to form $P(i+1)$
   d) go back to (a) with $P(i+1)$

3. Continue until some stopping criteria is satisfied.
   Report statistics!
Reproduction Methods

1. Cross-over

1\textsuperscript{st} Generation (parents)

\begin{align*}
\text{Parent 1} & \quad \text{Parent 2} \\
R P S S R P S & \quad S P P R R S P
\end{align*}

\begin{align*}
\text{2\textsuperscript{nd} Generation (offspring)} \\
S P P S R P S & \quad R P S R R S P
\end{align*}

2. Mutation

\begin{align*}
\text{R P S S R P S} \\
\downarrow \\
\text{R R S S R P P}
\end{align*}
Simulation Results:

A. Both are undounded:

B. One is bounded, the other is unbounded:

On the average, the result is a draw.

The bounded fellow is almost always defeated.
C. Both are bounded:

Player 1: Rock-Scissors
Player 2: Paper-Scissors-Lizard

Player 1 who has less info is getting higher payoff!
### 5-option universe

<table>
<thead>
<tr>
<th># of options of Player 1</th>
<th># of options of Player 2</th>
<th># of possible combinations</th>
<th># of solutions found</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 opt</td>
<td>5 opt</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>3 opt</td>
<td>5 opt</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>2 opt</td>
<td>5 opt</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>3 opt</td>
<td>4 opt</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
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<tr>
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<td>3 opt</td>
<td>100</td>
<td>10</td>
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Total # of possible combinations: 225
Total # of solutions found: 10

### 7-option universe

<table>
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<th># of options of Player 1</th>
<th># of options of Player 2</th>
<th># of possible combinations</th>
<th># of solutions found</th>
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</thead>
<tbody>
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<td>7 opt</td>
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<tr>
<td>5 opt</td>
<td>7 opt</td>
<td>21</td>
<td>0</td>
</tr>
<tr>
<td>4 opt</td>
<td>7 opt</td>
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<td>0</td>
</tr>
<tr>
<td>3 opt</td>
<td>7 opt</td>
<td>35</td>
<td>0</td>
</tr>
<tr>
<td>2 opt</td>
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<td>5 opt</td>
<td>6 opt</td>
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<td>0</td>
</tr>
<tr>
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<tr>
<td>2 opt</td>
<td>3 opt</td>
<td>735</td>
<td>98</td>
</tr>
</tbody>
</table>

Total # of possible combinations: 5509
Total # of solutions found: 287
Elimination of weakly dominated strategies:

Player 1

<table>
<thead>
<tr>
<th></th>
<th>Paper</th>
<th>Scissors</th>
<th>Lizard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>-1, 1</td>
<td>1, -1</td>
<td>1, -1</td>
</tr>
<tr>
<td>Scissors</td>
<td>1, -1</td>
<td>0, 0</td>
<td>1, -1</td>
</tr>
</tbody>
</table>
Lemma: In a zero-sum game with a full set of actions $\Omega$ for both players, any arbitrary deletion of an action $a_y \in \Omega$ by player $y$ makes a certain action $a_x \in \Omega$ by player $x$ weakly dominated.

Proof:

Let $a_y$ be the action deleted from the roster of actions of player $y$. Since there are $(n - 1)/2$ actions of $x$ that beats $a_y$, these actions $a_x^-$ will now have negative expected payoffs i.e.

$$a_x^- = \left\{ a_x \in \Omega_x \left| \sum_{y=1}^{n-1} u_x(a_x, a_y) P_y < 0 \right. \right\}.$$

To show that one of these $a_x^-$ is weakly dominated, pick an action $a'_x$ such that it can beat the same set of $y$ actions that a $a_x^-$ can defeat. There are two of these $a_x^-$ that can fulfill this: One that defeats $a'_x$ and another that is defeated by $a'_x$. By choosing the latter, we now have an $a_x^* \in a_x^-$ whose $u_x(a'_x, a_y) \geq u_x(a_x^*, a_y)$ for all $a_y \in \Omega \setminus a_y$. The action $a_x^*$ is therefore weakly dominated brought about by the removal of action $a_y$.
**Proposition 1:** A player (fully-rational) who employs all the possible actions in a finite universal set $\Omega$ always obtains a higher expected payoff than a player (bounded rational) who only employs a set of actions $A$, where $A \subset \Omega$.

**Proposition 2:** For any action set $A \subset \Omega$ that is available to player $x$, there exists an action set $B \subset \Omega$ available to player $y$ (where the number of elements in $B$ is less than the number of elements in $A$) such that the expected payoff it gives to $y$ is higher than what $x$ can obtain from employing $A$. 
The fellow with less options wins around 65 percent of the time!
The End

Thank you!